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TWO-DIMENSIONAL CONSOLIDATION OF NORMALLY CONSOLIDATED SOILS

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled TWO-DIMENSIONAL CONSOLIDATION OF NORMALLY CONSOLIDATED SOILS submitted by N. Ramanujam in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The non-linear theory of one-dimensional consolidation formulated by Davis and Raymond (1965) corresponds with experimental results for normally consolidated soils.

This thesis presents an analytical study of two-dimensional consolidation of normally consolidated soils. The assumptions adopted in the Davis and Raymond theory (1965) and the work of Henkel and Sowa (1963) on normally consolidated soils are used in this thesis to formulate general equations. The total stress components are derived from elastic stress distribution for a strip load on a half-space.

A general numerical method of solution is presented for thin layers in which total stress remains constant with respect to time and space.

Discussions are presented on the significance of non-elastic effects on the rate of dissipation of excess pore pressure. These discussions indicate that the rate of consolidation considered is almost independent of the pore pressure induced by shear stresses.

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CHAPTER I

INTRODUCTION

The theory of consolidation, as formulated by Terzaghi (1925), is based on the development and subsequent dissipation of excess pore pressure in a saturated soil medium. Consolidation is a process involving a time dependent volume reduction and associated decrease in moisture content. In deriving the differential equation governing one-dimensional consolidation, Terzaghi assumed constant coefficients of permeability and compressibility for the soil and small strains. All these simplifying assumptions make possible a linear theory and the governing equation facilitates closed form solution for given boundary conditions.

One-dimensional consolidation is due either to a lateral physical restraint, as in the oedometer test, or due to a surface loading which is of uniform magnitude and extends infinitely in all directions. Published field evidence of the rate at which foundations on clay settle suggests, that the actual rates are generally faster than those predicted by the one-dimensional consolidation theory. It is obvious that in many practical cases, the geometric conditions are not one-dimensional and that horizontal dissipation of pore pressure, which is not taken into account in the one-dimensional theory, increases rate of settlement. In spite of these limitations, the mathematical simplicity of one-dimensional theory permits a

large variety of solutions to be obtained. The use of three-dimensional theory for practical foundation problems appears to be hindered by a shortage of theoretical solutions covering an adequate range of practical situations.

The world is non-linear and contrary to many notions inherited from classical mechanics, the events that take place seldom are linear. There are no strictly homogeneous soils. All deformations, velocities and stress increments need not be small. There are no ideal fluids. This does not imply that all of these complexities of nature need be considered in practical designs, but the best design will always emerge from the most accurate assessments of natural condition. Terzaghi's linear theory relies on a set of unreal assumptions and is therefore only an approximation.

Non-linear behaviour in soil systems is usually placed into one of three categories.

1) Geometric non-linearity, which arises from non-linear terms in the kinematic equations,
2) Material non-linearity, which arises from non-linearities in the constitutive equations,
3) Combined geometric and material non-linearity.

Differences in observed and predicted results have stimulated many workers to introduce a series of modifications to Terzaghi's theory. Schiffman and Gibson (1964) studied the progress of consolidation in a stratum of clay through which there is a known continuous variation of the consolidation parameters. They have

presented a theory taking these changes into account. To overcome the inaccuracies incorporated in the Terzaghi theory, Davis and Raymond (1965) presented a small strain theory for compressible normally consolidated clay by taking into consideration a stress dependent coefficient of compressibility and coefficient of permeability. Barden and Berry (1965) also presented a small strain theory with a variation of consolidation parameters. Large strains have been considered by Mikasa (1965). Gibson et al (1967) developed a general theory of one-dimensional consolidation for which the limitation of small strains is not imposed and both the variation of soil compressibility and permeability during consolidation has been taken into account.

Unavoidably, many of the above modifications lead to nonlinear partial differential equations which render classical methods of mathematical analysis inapplicable. When such is the case, recourse is usually made to numerical methods for quantitative solutions to problems.

1.1 Terzaghi's Classical One-Dimensional Theory

For many field problems, the one-dimensional consolidation theory (Terzaghi, 1925) is sufficiently accurate. The basic assumptions involved in the theory are:

(i) the clay is saturated and homogeneous

(ii) the pore water and soil grains are incompressible in comparison with the soil skeleton

(iii) the flow and consolidation are one-dimensional

(iv) the fluid flow follows Darcy's Law

(v) the coefficients of permeability and compressibility are constant during the process of consolidation

(vi) strains in the soil skeleton are controlled exclusively by the effective stress via a linear time independent relation

(vii) strains, velocities and stress increments are small.

The basic theory considers only the diffusion of water through a porous medium. Equating the volume of pore water squeezed out with the reduction in void ratio of an element of the soil, the well known linear equation of Terzaghi one-dimensional consolidation is deduced. The governing equation may be written as

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \qquad 1.1$$

where C_v denotes coefficient of consolidation and

u denotes pore water pressure.

The application of one-dimensional theory, which relies on a set of unreal assumptions concerning material properties and the dimensionality of compression, paradoxically, yields acceptable results and permits a wide range of solutions to be obtained. An examination of Equation 1.1 reveals that it is linear and closed form solutions are possible. An important feature of the linearity is, the additive, or superposable, nature of solutions. Although Equation 1.1 describes the behaviour of the soil in an artificial

way, its superposition properties and closed form solutions render it both attractive and useful, provided the inherent limitations are observed.

Because of linearity it is appropriate to normalize the various parameters to make the equation non-dimensional. In this way, the solution obtained in terms of dimensionless parameters is in a more suitable form for general application. Recent developments in numerical analysis and the modern advances in computer technology provide improved capability of achieving greater versatility in handling one-dimensional consolidation equations.

1.2 Terzaghi-Rendulic Two-Dimensional Theory

When a soil consolidates under an applied load, compression and drainage often take place in three dimensions rather than in one as assumed in Terzaghi's one-dimensional theory. However, the existing knowledge of the consolidation characteristics of soil in three-dimensions is still in the preliminary stages. Since the displacement of principal importance in structural applications is vertical, with due reservations, the influence of other than vertical directions may often be neglected. However, the process of drainage of water in three dimensions does have an effect on the rate of settlement of a soil loaded at the surface.

The simple consolidation theory generally attributed to Terzaghi (1925) and Rendulic (1937) considers the three dimensional diffusion of water through a porous medium. The conventional theory of consolidation, as formulated by Terzaghi (1925), assumes that

only the excess pore pressure contributes to the progress of consolidation. For a time-independent loading the progress of consolidation is governed by a single relationship of heat conduction type as Equation 1.1.

In conventional theory the internal total stresses are generated only by the applied loads. This interpretation has led to the Terzaghi-Rendulic (pseudo three-dimensional) theory. For a time-independent loading, the excess pore pressures are governed by

$$C_p \nabla^2 u = \frac{\partial u}{\partial t} \tag{1.2}$$

where C_p denotes coefficient of consolidation.

The formulation of pseudo three-dimensional theory involves theoretical inconsistencies as it takes no account of strain compatability within the soil mass. As with one-dimensional theory, during the process of consolidation, the total stress which may be calculated from elastic stress distribution theory, remains constant for time-independent loading.

Equation 1.2 is a half-way point between the one-dimensional theory and a true three-dimensional theory. This equation retains all the characteristics of one-dimensional theory and is based on the same assumptions. Equation 1.2 provides a means for dissipation of excess pore pressure in all directions but the consolidation is restricted to the vertical direction only.

Under two-dimensional drainage conditions Equation 1.2

becomes

$$C_p \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\partial u}{\partial t} \qquad 1.3$$

Equation 1.3 governing the rate of dissipation is of the heat conduction type and may be used for all types of problems solved using the classical one-dimensional theory. Since Equation 1.3 has all the characteristics of the classical one-dimensional theory, it is called the Terzaghi equation for consolidation in two-dimensions.

1.3 Non-Linear Consolidation Theories

To improve correspondence between prediction and performance, many of the assumptions employed in Terzaghi's theory have been modified by various workers from time to time. Modest deviations from the basic theory yield non-linear partial differential equations which render classical methods of mathematical analysis inapplicable. Complexities of non-linear analysis have created a "non-linear barrier" which, only in very recent times, has begun to be penetrated by means of the efficient use of modern high speed computers. Faced with the necessity of obtaining solutions, when classical methods show sign of breakdown, recourse is usually made to numerical techniques.

Finite Strain Non-Linear Theory due to Gibson et al (1967)

Gibson et al (1967) published a general theory of one-dimensional consolidation for which the limitations of small strains

are not imposed, and the variation of compressibility and permeability during consolidation has been taken into account. The validity of Darcy's Law has been assumed but it has been recast in a form in which the relative velocity of soil skeleton and the pore fluid related to the excess pore pressure gradient.

For the particular case of a horizontal thin layer, the self weight effect is negligible and the governing equation in terms of the void ratio, e, is

$$\frac{\partial}{\partial a}\left[C_F \frac{\partial e}{\partial a}\right] = \frac{\partial e}{\partial t} \qquad 1.4$$

where "a" is the vertical distance of an element of the soil skeleton from an embedded datum plane (Fig. 1.1). The quantity C_F is closely related to the familiar coefficient of consolidation C_V of Terzaghi's theory, by the expression

$$C_F = \left[\frac{1 + e_o}{1 + e}\right] C_V \qquad 1.5$$

By assuming C_F to remain constant during consolidation, the differential Equation 1.4 may be linearized under a specific pressure increment, so that

$$C_F \frac{\partial^2 e}{\partial a^2} = \frac{\partial e}{\partial t} \qquad 1.6$$

The important feature of Equation 1.6 is that, unlike

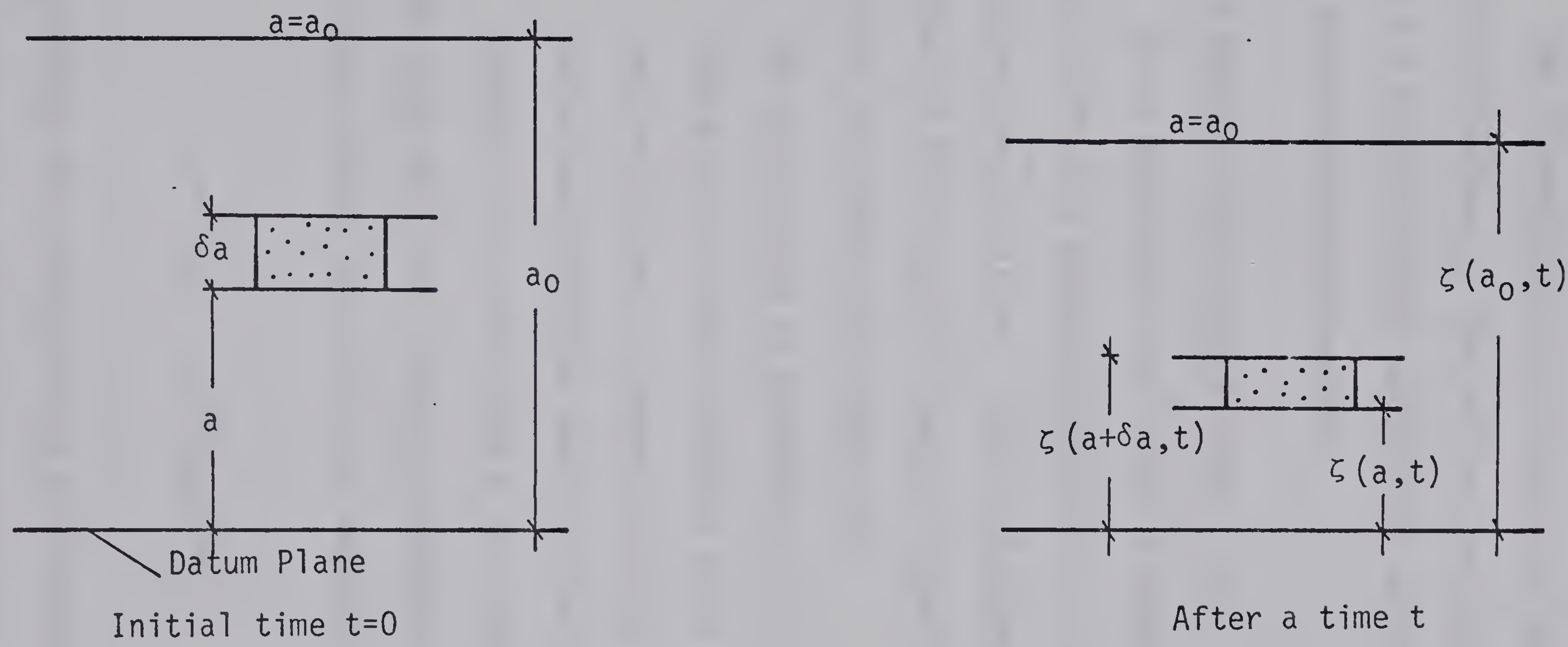


FIG. 1.1 CONFIGURATION OF SOIL ELEMENT

the corresponding Terzaghi equation, it holds without any restriction on the form of relation between effective pressure and void ratio.

The linearization procedure may lead to errors, either because of the nature of the soil or the size of the load increment. Equation 1.4 must be solved numerically using a relation between C_F and e determined experimentally.

Davis and Raymond's Non-Linear Theory

It is known that the Terzaghi theory does not hold for the case of soft clay whose permeability, k, and compressibility, m_v, change during consolidation. Davis and Raymond (1965) introduced a modification to the Terzaghi theory in order to overcome the inaccuracies of the following assumptions:

(i) the coefficient of permeability, k, is a constant during consolidation under a given stress increment; and

(ii) the coefficient of compressibility, m_v, remains constant during consolidation under a given stress increment.

Instead of assuming that m_v is constant, they adopt the well established empirical linear relationship between void ratio, e and the logarithm of the effective pressure, σ' as

$$e = e_o - I_c \log_{10} \left(\frac{\sigma'}{\sigma'_o}\right) \qquad 1.7$$

where e_o denotes the void ratio of soil subjected to σ'_o

σ'_o denotes the effective pressure at point "o" on the e - log σ' curve

and I_c denotes the compression index of a soil and is constant. The coefficient of compressibility, m_v of the soil skeleton is given by

$$m_v = - \frac{1}{(1+e)} \frac{\partial e}{\partial \sigma'} \qquad 1.8$$

Differentiating Equation 1.7 with respect to effective pressure, σ', and substituting in Equation 1.8, yields

$$m_v = \frac{0.434 \, I_c}{(1+e) \, \sigma'} \qquad 1.9$$

During the consolidation process the total volume (1+e) varies less with time than the effective pressure, σ', hence (1+e) may be considered constant for any load increment. With this assumption Equation 1.9 becomes

$$m_v = \frac{A}{\sigma'} \qquad 1.10$$

where A is a constant which depends on the initial void ratio and compressibility index, I_c.

Davis and Raymond point out that during any given pressure increment the coefficient of consolidation, C_V, usually changes much less than the coefficient of compressibility, m_v, for normally consolidated clay. They assumed that C_V remains constant, i.e.,

$$C_V = \frac{k}{m_V \gamma_w} = \text{constant} \qquad 1.11$$

where γ_w denotes unit weight of water.

This can be seen to be equivalent to assuming that as the soil particles move closer together the decrease in permeability, k, is proportional to the decrease in compressibility, m_V.

By adopting Equations 1.10 and 1.11 in place of the assumptions (i) and (ii) in the Terzaghi theory, Davis and Raymond derived the following governing equation which is

$$\frac{1}{\sigma'} \frac{\partial \sigma'}{\partial t} = - C_V \left[\frac{1}{\sigma'} \frac{\partial^2 u}{\partial z^2} \; 0 \; \frac{1}{(\sigma')^2} \frac{\partial u}{\partial z} \frac{\partial \sigma'}{\partial z}\right] \qquad 1.12$$

Equation 1.12 is a weakly non-linear equation for a normally consolidated soil under a constant load. Using the substitution,

$$w = \log_{10} \frac{\sigma'}{\sigma'_f} = \log_{10} \frac{\sigma'_f - u}{\sigma'_f} \qquad 1.13$$

where σ'_f denotes final effective pressure, and differentiating this expression with respect to z and t and substituting in Equation 1.12 yields a simpler version of the differential equation in terms of the function, w

$$C_V \frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial t} \qquad 1.14$$

This equation is identical in form to that of the Terzaghi linear theory and a solution to the equation will be unique depending on the specified boundary and initial conditions.

Modification by Barden and Berry (1965)

Barden and Berry (1965) modified the assumptions in Terzaghi's theory to agree more closely with the physical properties of such materials as clays which exhibit marked secondary consolidation (creep) effects. The modifications are:

(i) the effects of structural viscosity which controls creep require that void ratio be a function of both effective stress and time,

(ii) for normally consolidated clays, the void ratio-logarithm of effective stress relation is a better approximation,

(iii) a void ratio-logarithm of permeability relation is considered.

Combining e-logk relation with the e-logσ' relation a general relation for permeability may be developed

$$k = k_f \, (1 + bu^n) \qquad 1.15$$

where k_f denotes final permeability
and b and n are constants.

Satisfying the equation of continuity for one-dimensional vertical flow in a saturated soil, Barden and Berry have presented

a small strain theory with the following non-linear equation

$$\frac{\partial}{\partial z}\left[\frac{k_f}{\gamma_w}(1 + bu^n)\frac{\partial u}{\partial z}\right] = \frac{C_c}{(1+e_o)}\frac{1}{(\sigma'_f - u)}\frac{\partial u}{\partial t} \qquad 1.16$$

Exact solutions to Equation 1.16 are difficult to obtain, hence finite difference solutions have been developed.

1.4 Scope of the Study

The non-linear theory of Davis and Raymond (1965) is in excellent agreement with experimental results for large as well as small pressure increment ratios (Burland and Roscoe, 1969). In engineering practice plane flow of pore water is assumed when analysing dissipation of pore water, for example, in rolled filled layers of earth dams or strip foundations, or embankments. An analytical approach is made to study the two-dimensional consolidation of normally consolidated soils (a strip load of width 2B of uniform intensity q resting on the half-plane). Parameters for normally consolidated soil considered herein are likely to arise in practice.

CHAPTER II

TWO-DIMENSIONAL CONSOLIDATION OF NORMALLY CONSOLIDATED SOILS

GENERAL APPROACH

2.1 Method of Analysis

Some of the well known and generally accepted principles of soil mechanics and foundation engineering are employed in this study. Classical theory of elasticity is used which assumes that a linear relation holds between stress and strain and that strains are small. Numerical techniques are utilized in conjunction with the assumptions used in the one-dimensional Davis and Raymond non-linear theory.

2.2 Assumptions Employed

The following physical, mathematical and engineering assumptions are made in the study.

(a) Normally consolidated, completely saturated soil.

(b) Darcy's law is valid.

(c) Problem is one of plane strain.

(d) Bishop's (1966) relationship is used to evaluate the intermediate principal stress, σ_2.

(e) Load increment is applied instantaneously.

(f) Initial excess pore pressures can be expressed in terms of octahedral normal stresses, octahedral shear stresses and the Skempton pore pressure parameter, A.

(g) Volume change is mainly due to hydrostatic stress.

(h) Empirical relationship between void ratio, e, and logarithm of the average effective stress, θ', (Henkel and Sowa, 1963) is valid.

(i) In spite of the variation of the coefficient of compressibility and permeability with the changes in stress, their combined effect is balanced to maintain the coefficient of consolidation constant.

(j) Soils are homogeneous.

(k) The coefficient of lateral earth pressure is constant and equal to the "at rest" value (k_o).

(l) Representative values for stresses are obtained for a strip load using the Bousinesq relations.

(m) The coefficient of consolidation is independent of the geometry of the loading and the boundary conditions.

(n) During the consolidation process, the load distribution on the surface of the soil layer and the associated total stress distribution in the soil mass are unaffected.

(o) The ground water table is at the upper surface of the soil layer.

2.3 Boundary Conditions

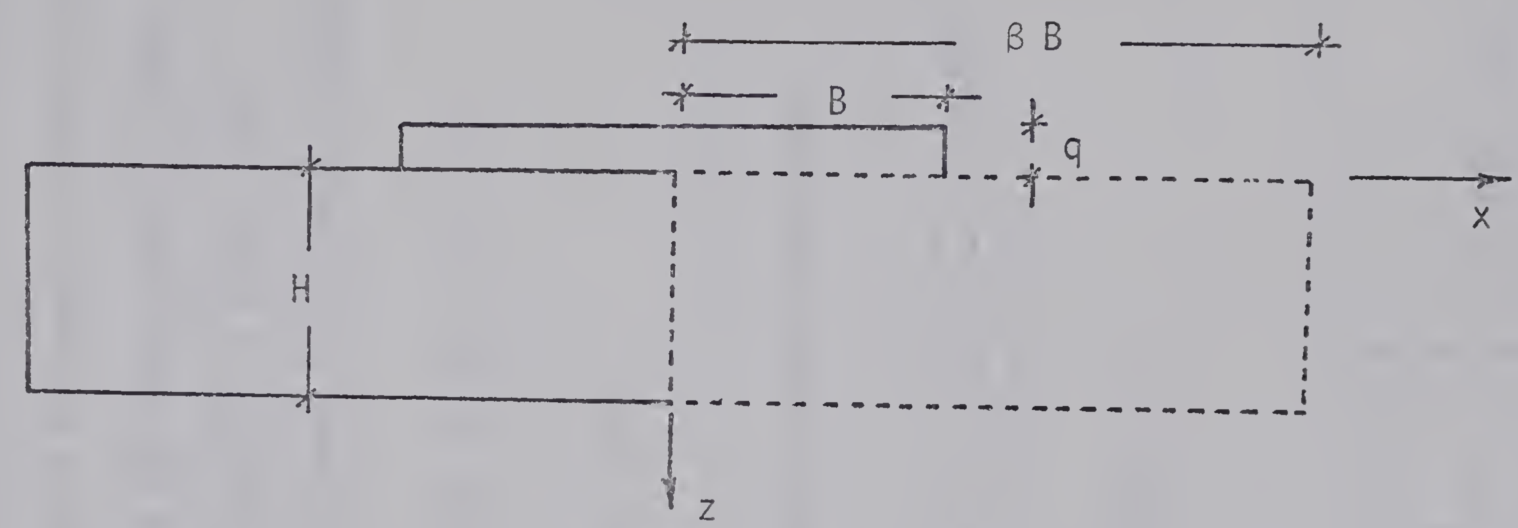
In common with the usual practice employed when analysing, for example, rolled filled layers of earth dams or strip foundations or embankments, two-dimensional flow of pore water is considered. The excess pore pressure, u, at the previous boundary surfaces (Fig. 2.1) vanishes and at the impervious boundary surfaces (an insulated boundary) the normal derivative of excess pore pressure vanishes. Due to symmetry only a half of the cross section need be considered.

MATHEMATICAL FORMULATION

2.4 Total Stress Distribution

Changes in stresses take place when a load is applied to the surface of the soil mass. There is a measurable total stress increase at all points within a "zone of influence". In the case of clay soils, it is of interest to compute the instantaneous excess pore water pressure distribution due to the applied load. With time, this fluid stress will dissipate, throwing increasing amounts of the applied stress onto the soil skeleton in the form of effective stress. Based on the assumptions employed in linear elasticity and referring to the co-ordinate system shown in Fig. 2.1 the induced stresses σ_x, σ_z and τ_{xz} acting at a point may be determined for a strip load resting on the surface.

The total induced stresses at a point, [S] within the compressible soil layer may be separated into dilatational and de-



$x=0$	$0 \leq x \leq B$	$x=\beta B$	$B \leq x \leq \beta B$	$0 \leq x \leq \beta B$
$0 \leq z \leq H$	$z=0$	$0 \leq z \leq H$	$z=0$	$z=H$
$\partial u/\partial x=0$	$u=0$	$u=0$	$u=0$	$\partial u/\partial z=0$

FIG. 2.1 GEOMETRY OF THE PROBLEM AND THE BOUNDARY CONDITIONS CONSIDERED

viatoric components [D'] and [D''] respectively and written as

$$[S] = [D'] + [D''] = \begin{bmatrix} \theta & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & \theta \end{bmatrix} + \begin{bmatrix} \sigma_x - \theta & 0 & \tau_{xz} \\ 0 & \sigma_y - \theta & 0 \\ \tau_{xz} & 0 & \sigma_z - \theta \end{bmatrix} \tag{2.1}$$

where $$\theta = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right) \tag{2.2}$$

and $$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\,\tau_{xz}^2} \tag{2.3}$$

For plane strain problems

$$\tau_{xy} = \tau_{yz} = 0 \tag{2.4}$$

and $$\sigma_y = 0.5 \cos^2 \phi' \,(\sigma_x + \sigma_z) \text{ (Bishop, 1966)} \tag{2.5}$$

2.5 Initial Excess Pore Pressure Distribution

When a load is applied to the saturated soil mass, the entire load is taken up by the water at the instant of loading. The pore pressure generates a pressure equal to the applied load. The process of consolidation depends upon the rate at which excess pore pressures are dissipated. One very important factor which governs this dissipation process, is the initial excess pore pressure distribution in the soil.

The theory of elasticity can be used to obtain the initial distribution of excess pore pressure in the clay soil. The pore pressure distribution introduces spatial gradients in the water which give rise to body forces in the soil.

For an ideal saturated soil with an elastic skeleton, the initial pore pressure, u_i, is given by

$$u_i = \theta \qquad 2.6$$

Soils dilate while undergoing volume changes as a result of shearing strains without variation in the hydrostatic stress. Henkel (1960) considered such an effect by modifying Equation 2.6 and it is generally accepted that the following equation is a good approximation.

$$u_i = \theta + a\,\tau_{oct} \qquad 2.7$$

where "a" is the Henkel pore pressure parameter which is related to the Skempton (1954) "A" parameter by the relation

$$\sqrt{2}a = A - \frac{1}{3} \qquad 2.8$$

The convenience of using θ, which includes no shearing effects, and τ_{oct}, which is independent of hydrostatic stress, is well demonstrated in Equation 2.7.

2.6 Governing Equation

Adopting the empirical relation between void ratio, e, and hydrostatic effective stress, θ', (Henkel and Sowa, 1963), as

$$e = e_o - N \log_{10} \frac{\theta'}{\theta'_i} \qquad 2.9$$

the coefficient of compressibility, m_v', is given by

$$m_v' = - \frac{1}{(1+e)} \frac{\partial e}{\partial \theta'} = \frac{0.434\ N}{(1+e)\ \theta'} \qquad 2.10a$$

where $$\theta'_i = (\sigma'_{xi} + \sigma'_{yi} + \sigma'_{zi})/3 \ , \qquad 2.10b$$

initial effective stress

$$\theta' = (\sigma'_x + \sigma'_y + \sigma'_z)/3 \qquad 2.10c$$

and N denotes the slope of e-logθ' curve.

During the consolidation process, (1+e) varies with time far less than θ' hence, (1+e) may be considered constant for any load increment. With this assumption Equation 2.10a becomes

$$m_v' = \frac{B}{\theta'} \qquad 2.10d$$

where $$B = \frac{0.434\ N}{(1+e)} = \text{a constant} \qquad 2.10e$$

For normally consolidated clay, Davis and Raymond pointed out that the coefficient of consolidation, C_V, varies much less than the coefficient of compressibility, m_v. They assumed C_V to be a constant. It can be therefore assumed that

$$C_V' = \frac{k}{\gamma_w m_v'} = \text{constant} \qquad 2.11$$

where k denotes the coefficient of permeability

γ_w denotes the unit weight of water

and C_V' denotes the coefficient of consolidation.

Using Darcy's law

$$v_z = k_z i_z = - \frac{k_z}{\gamma_w} \frac{\partial u}{\partial z} \qquad 2.12$$

where v_z denotes the velocity of flow in the z-direction

k_z denotes the coefficient of permeability in the z-direction

i_z denotes the hydraulic gradient in the z-direction

and u denotes excess pore pressure

and substituting Equation 2.10d in Equation 2.11 the resulting equation is

$$\frac{k_z}{\gamma_w} = \frac{B}{\theta'} C_V' \qquad 2.13$$

Substituting Equation 2.13 in Equation 2.12 and differentiating the resulting equation with respect to z yields

$$\frac{\partial v_z}{\partial z} = - \frac{\partial}{\partial z} \left[\frac{B\ C_V'}{\theta} \frac{\partial u}{\partial z}\right] \quad 2.14a$$

$$= - B\ C_V' \frac{\partial}{\partial z} \left[\frac{1}{\theta'} \frac{\partial u}{\partial z}\right] \quad 2.14b$$

$$= - B\ C_V' \left[\frac{1}{\theta'} \frac{\partial^2 u}{\partial z^2} - \frac{1}{(\theta')^2} \frac{\partial \theta'}{\partial z} \frac{\partial u}{\partial z}\right] \quad 2.14c$$

in the same manner similar expressions such as Equation 2.14c can be produced for the x and y directions. These are,

$$\frac{\partial v_x}{\partial x} = - B\ C_V' \left[\frac{1}{\theta'} \frac{\partial^2 u}{\partial x^2} - \frac{1}{(\theta')^2} \frac{\partial \theta'}{\partial x} \frac{\partial u}{\partial x}\right] \quad 2.15$$

$$\frac{\partial v_y}{\partial y} = - B\ C_V' \left[\frac{1}{\theta'} \frac{\partial^2 u}{\partial y^2} - \frac{1}{(\theta')^2} \frac{\partial \theta'}{\partial y} \frac{\partial u}{\partial y}\right] \quad 2.16$$

The rate of water lost from an element dxdydz is given by,

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) dxdydz$$

$$= - B\ C_V' \left\{\frac{1}{\theta'} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{1}{(\theta')^2} \left[\frac{\partial \theta'}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \theta'}{\partial y} \frac{\partial u}{\partial y}\right.\right.$$

$$\left.\left. + \frac{\partial \theta'}{\partial z} \frac{\partial u}{\partial z}\right]\right\} dxdydz \quad 2.17$$

The strain developed within the soil element is given by

$$\varepsilon = \frac{(e_o - e)}{(1+e_o)} \quad 2.18a$$

where ε denotes strain

and e_o denotes the void ratio corresponding to zero strain and to the stress θ_i'.

Making use of Equation 2.9, strain can be rewritten as

$$\varepsilon = \frac{N}{(1+e_o)} \log_{10} \left(\frac{\theta'}{\theta'_i}\right) \qquad 2.18b$$

The rate of volume change can be obtained by differentiating the strain with respect to time

$$\frac{\partial \varepsilon}{\partial t} = \frac{N}{(1+e_o)} \frac{0.434}{\theta'} \frac{\partial \theta'}{\partial t} \, dxdydz \qquad 2.19a$$

Considering the change in void ratio, e, is too small to affect the total volume, (1+e), it may be assumed to be constant. Hence the Equation 2.19a may be rewritten, making use of the Equation 2.10e, as

$$\frac{\partial \varepsilon}{\partial t} = \frac{B}{\theta'} \frac{\partial \theta'}{\partial t} \, dxdydz = m_v' \frac{\partial \theta'}{\partial t} \, dxdydz \qquad 2.19c$$

Equations of continuity demand that the amount of water lost must be equal to the reduction in volume. Therefore

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) dxdydz = \frac{\partial \varepsilon}{\partial t} \, dxdydz \qquad 2.20$$

Equating Equations 2.17 and 2.19c to satisfy continuity,

the resulting equation becomes

$$\frac{1}{\theta'}\frac{\partial\theta'}{\partial t} = - C_V'\{ \frac{1}{\theta'}(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2})$$

$$- \frac{1}{(\theta')^2}[\frac{\partial\theta'}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial\theta'}{\partial y}\frac{\partial u}{\partial y} + \frac{\partial\theta'}{\partial z}\frac{\partial u}{\partial z}]\} \qquad 2.21$$

Equation 2.21 is the general three-dimensional consolidation equation for the assumptions employed in Davis and Raymond theory.

The original Davis and Raymond one-dimensional equation (no flow in the x-direction and the y-direction) may be derived by considering $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$ and $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = 0$. Equation 2.21 then becomes

$$\frac{1}{\sigma'_z}\frac{\partial\sigma'_z}{\partial t} = - C_V\{\frac{1}{\sigma'_z}\frac{\partial^2 u}{\partial z^2} - \frac{1}{(\sigma'_z)^2}[\frac{\partial\sigma'_z}{\partial t}\frac{\partial u}{\partial z}]\} \qquad 2.22$$

which is the same as given by Davis and Raymond (1965).

For two-dimensional consolidation (flow in the y-direction is zero), it can be considered that $\frac{\partial u}{\partial y} = 0$ and $\frac{\partial^2 u}{\partial y^2} = 0$ and the governing equation becomes

$$\frac{\partial\theta'}{\partial t} = - C_V'\{(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}) - \frac{1}{\theta'}(\frac{\partial\theta'}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial\theta'}{\partial z}\frac{\partial u}{\partial z})\} \qquad 2.23$$

For a constant load with time, $(\sigma_x + \sigma_y + \sigma_z)/3 = \theta$ can be considered as constant, hence

$$\frac{\partial \theta}{\partial t} = 0 \tag{2.24}$$

By definition

$$\theta = \theta' + u \tag{2.25}$$

and differentiating Equation 2.25 and substituting in Equation 2.24 yields

$$\frac{\partial \theta'}{\partial t} = - \frac{\partial u}{\partial t} \tag{2.26}$$

The stress components σ_x, σ_z and τ_{xz} for the soil medium (Fig. 2.1), are (Harr, 1966)

$$\sigma_x = \frac{q}{\pi} [\tan^{-1} \frac{x+B}{z} - \tan^{-1} \frac{x-B}{z} - \frac{2zB(x^2-z^2-B^2)}{(x^2+z^2-B^2)^2+4B^2z^2}] \tag{2.27}$$

$$\sigma_z = \frac{q}{\pi} [\tan^{-1} \frac{x+B}{z} - \tan^{-1} \frac{x-B}{z} + \frac{2zB(x^2-z^2-B^2)}{(x^2+z^2-B^2)^2 + 4B^2z^2}] \tag{2.28}$$

and

$$\tau_{xz} = \frac{4q}{\pi} [\frac{Bxz^2}{(x^2+z^2-B^2)^2 + 4B^2z^2}] \tag{2.29}$$

$$z > 0, \ |x| < \infty$$

where q denotes intensity of load

and B denotes half of the loaded width.

Using the Equations 2.5, 2.25, 2.26, 2.27, 2.28 and substituting in Equation 2.23 yields

$$\frac{\partial u}{\partial t} = C_V' \left\{ \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{1}{(\theta'_f - u)} \left\{ \left[C_1\left(\frac{z}{z^2+(x+B)^2} - \frac{z}{z^2+(x-B)^2} - \frac{\partial u}{\partial x}\right) \frac{\partial u}{\partial x}\right] + C_1\left[\frac{(x-B)}{z^2+(x-B)^2} - \frac{(x+B)}{z^2+(x+B)^2} - \frac{\partial u}{\partial z}\right] \frac{\partial u}{\partial z}\right\} \right\} \qquad 2.30$$

where $C_1 = \frac{2}{3\pi} q[1 + 0.5 \cos^2\phi']$.

Equation 2.30 is the general governing equation for consolidation of a normally consolidated soil with two dimensional drainage. This equation is highly non-linear and could not be solved with the numerical methods adopted in this study.

Assuming a constant load with respect to time as well as with space (as in the case of thin clay deposits) and differentiating Equation 2.25 with respect to time and space and substituting in Equation 2.23, yields

$$\frac{\partial u}{\partial t} = - C_V' \left\{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{1}{\theta'} \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]\right\} \qquad 2.31$$

Equation 2.31 represents two-dimensional consolidation of normally consolidated soils under a constant load. A solution to Equation 2.31 will be unique depending on specified boundary and initial conditions.

Using the substitution

$$w = \log_{10} \frac{\theta'}{\theta'_f} = \log_{10} \frac{\theta'_f - u}{\theta'_f} \qquad 2.32a$$

where $$\theta'_f = \left(\frac{\sigma'_{xf} + \sigma'_{yf} + \sigma'_{zf}}{3}\right) \qquad 2.32b$$

= final effective stress

and differentiating with respect to x

$$\frac{\partial w}{\partial x} = - \frac{0.434}{\theta'} \frac{\partial u}{\partial x} \qquad 2.33$$

$$\frac{\partial^2 w}{\partial x^2} = - 0.434 \left[\frac{1}{\theta'} \frac{\partial^2 u}{\partial x^2} + \frac{1}{(\theta')^2} \left(\frac{\partial u}{\partial x}\right)^2\right] \qquad 2.34$$

Similarly,

$$\frac{\partial w}{\partial z} = - \frac{0.434}{\theta'} \frac{\partial u}{\partial z} \qquad 2.35$$

$$\frac{\partial^2 w}{\partial z^2} = - 0.434 \left[\frac{1}{\theta'} \frac{\partial^2 u}{\partial z^2} + \frac{1}{(\theta')^2} \left(\frac{\partial u}{\partial z}\right)^2\right] \qquad 2.36$$

Differentiating Equation 2.32a with respect to time, yields

$$\frac{\partial w}{\partial t} = \frac{0.434}{\theta'} \frac{\partial \theta'}{\partial t} \qquad 2.37$$

Substituting Equations 2.34, 2.36 and 2.37 into Equation 2.31 gives the simpler version of the differential equation in terms of the function, w

$$\frac{\partial w}{\partial t} = C_V' \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right] \qquad 2.38$$

Equation 2.38 is identical to that of the Terzaghi Equation 1.3, and can be solved in the same way.

For the problem considered herein (a strip load of width, 2B, of uniform load intensity, q, resting on a half-space) the boundary conditions are similar in terms of u and w (Fig. 2.1). These conditions are given in Table 1.

TABLE 1

SIMILARITY OF THE BOUNDARY CONDITIONS DEFINED IN TERMS OF u OR w

$t = 0$	and	$0 \leq z \leq H$	and	$0 \leq x \leq \beta B$	and	$u = \theta'_f - \theta'_i$	then	$w = \log_{10} \frac{\theta'_i}{\theta'_f}$
$0 \leq t \leq \propto$	"	$z = H$	"	$0 \leq x \leq \beta B$	"	$\frac{\partial u}{\partial z} = 0$	"	$\frac{\partial w}{\partial z} = 0$
$0 \leq t \leq \propto$	"	$0 \leq z \leq H$	"	$x = 0$	"	$\frac{\partial u}{\partial x} = 0$	"	$\frac{\partial w}{\partial x} = 0$
$0 \leq t \leq \propto$	"	$z = 0$	"	$0 \leq x \leq \beta B$	"	$u = 0$	"	$w = 0$
$t = \propto$	"	$0 \leq z \leq H$	"	$0 \leq x \leq \beta B$	"	$u = 0$	"	$w = 0$

CHAPTER III

ANALYSIS BY NUMERICAL METHODS

3.1 General

Closed-form solutions of the non-linear equation for specified boundary and initial conditions are difficult to obtain and alternate methods must often be used, for example, numerical methods.

The expansion of the function by Taylor's series provides the fundamental concept in the finite difference approximation of a differential equation. By representing the derivatives as finite differences, a system of differential equations is converted to a system of difference equations and the problem reduces to the solution of a set of simultaneous equations.

Most numerical schemes call for step by step procedures. In the process of solution of differential equations, errors are introduced at each step of the calculation due to the approximations in finite difference and numerical calculations. The finite difference scheme must be so formulated that the error growth with each step is tolerable. If error grows too rapidly, the scheme is unstable.

3.2 Alternating-Direction Implicit Method (ADI)

The alternating-direction implicit (ADI) method was introduced by Peaceman and Rachford (1955) for use in problems involving a large number of internal nodal points. In this method the size of the matrix to be solved at each time step is reduced at the expense of solving the reduced matrix many times for each time step.

An alternating-direction implicit procedure requires a line-by-line solution of small sets of simultaneous equations. These equations may be solved by a non-iterative direct method. Direct methods for solving sets of equations on a computer are usually based on Gaussian elimination.

The finite difference representation for the two-dimensional consolidation equation,

$$\frac{\partial u}{\partial T} = \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial x^2} \qquad 3.1$$

in the ADI method (Fig. 3.1) is given by

$$\{u(L,M,N+1) - u(L,M,N)\}/\Delta T$$

$$= \{u(L-1,M,N+1) - 2u(L,M,N+1) + u(L+1,M,N+1)\}/(\Delta z)^2$$

$$+ \{u(L,M-1,N) - 2u(L,N,N) + u(L,N+1,N)\}/(\Delta x)^2 \qquad 3.2$$

This is used to advance the solution from the N to the (N+1) time

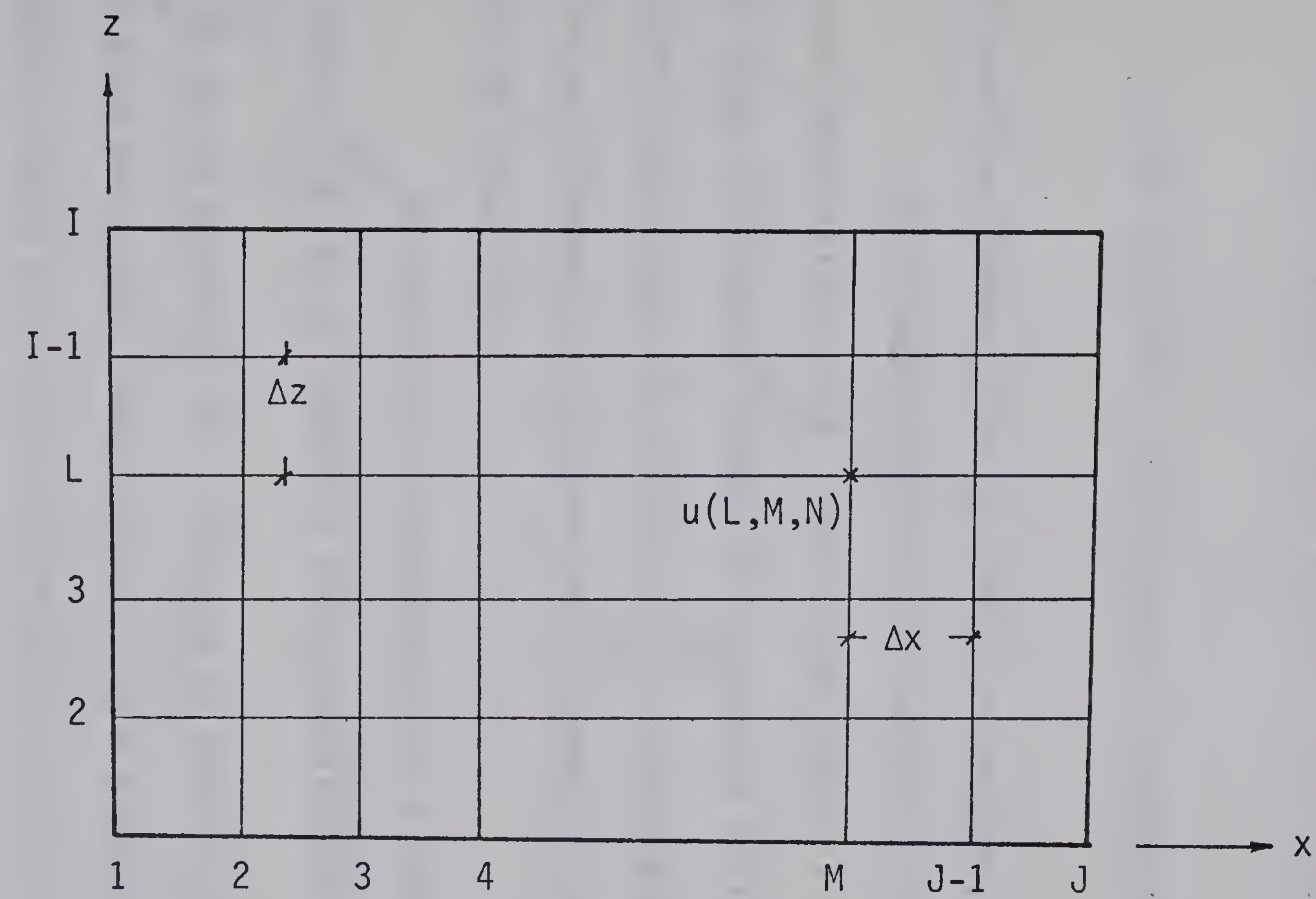


FIG. 3.1 FINITE DIFFERENCE SCHEME (ADI)

step and the equation,

$$\{u(L,N,N+2) - u(L,N,N+1)\}/\Delta T$$

$$= \{u(L-1,N,N+1) - 2u(L,N,N+1) + u(L+1,M,N+1)\}/(\Delta z)^2$$

$$+ \{u(L,M-1,N+2) - 2u(L,M,N+2) + u(L,M+1,N+2)\}/(\Delta x)^2 \qquad 3.3$$

is used to advance from the (N+1) to the(N+2) time step.

This method consists of replacing only one of the second-order derivatives, $\frac{\partial^2 u}{\partial z^2}$, say, by an implicit difference approximation in terms of unknown values of u from the (N+1) time-level, the other second-order derivative, $\frac{\partial^2 u}{\partial x^2}$, being replaced by an explicit finite difference approximation in terms of known values of u from the Nth time-level.

The solution is advanced to the (N+2) time-level by replacing $\frac{\partial^2 u}{\partial x^2}$ by an implicit finite-difference approximation and $\frac{\partial^2 u}{\partial z^2}$ by an explicit one. The use of each of the above equations at each time step leads to P sets of P simultaneous equations. Because these are solved alternately by sets of rows and sets of columns, the method may be considered as a line method with alternating directions.

3.3 Finite Difference Representation

Using the non-dimensional variables,

$$U = \frac{u}{\theta'_i} \qquad 3.4$$

$$Z = \frac{z}{H} \qquad 3.5$$

$$X = \frac{x}{H} \qquad 3.6$$

and

$$T = \frac{C_V' t}{H^2} = \text{time factor} \qquad 3.7$$

Equation 2.31 can be rewritten as

$$\frac{\partial U}{\partial T} = \{(\frac{\partial^2 U}{\partial Z^2} + \frac{\partial^2 U}{\partial X^2}) + \frac{1}{(\frac{\theta'_f}{\theta'_i} - U)} [(\frac{\partial U}{\partial Z})^2 + (\frac{\partial U}{\partial X})^2]\} \qquad 3.8a$$

Defining $\frac{\theta'_f}{\theta'_i}$ = DR (stress increment ratio) Equation 3.8a becomes

$$\frac{\partial U}{\partial T} = \{(\frac{\partial^2 U}{\partial Z^2} + \frac{\partial^2 U}{\partial X^2}) + \frac{1}{(DR-U)} [(\frac{\partial U}{\partial Z})^2 + (\frac{\partial U}{\partial X})^2]\} \qquad 3.8b$$

The governing Equation 3.8b can be expressed in finite differences (making use of the ADI method) as

$$\{U(L,M,N+1) - U(L,M,N)\}/\Delta T$$

$$= \{U(L-1,M,N+1) - 2U(L,M,N+1) + U(L+1,M,N+1)\}/(\Delta Z)^2$$

$$+ \{U(L,M-1,N) - 2U(L,M,N) + U(L,M+1,N)\}/(\Delta X)^2$$

$$+ \{1/[DR(L,M) - U(L,M,N)]\} \{[(U(L+1,M,N) - U(L-1,M,N))/2\Delta Z]^2$$

$$+ [(U(L,M+1,N) - U(L,M-1,N))/2\Delta X]^2\}$$

and for the next time step, as

$$U(L,M,N+2) - U(L,M,N+1) /\Delta T$$

$$= \{U(L-1,M,N+1) - 2U(L,M,N+1) + U(L+1,M,N+1)\}/(\Delta Z)^2$$

$$+ \{U(L,M-1,N+2) - 2U(L,M,N+2) + U(L,M+1,N+2)\}/(\Delta X)^2$$

$$+ \{1/[DR(L,M) - U(L,M,N+1)]\}\{[(U(L+1,M,N+1) - U(L-1,M,N+1))/2\Delta Z]^2$$

$$+ [(U(L,M+1,N+1) - U(L,M-1,N+1))/2\Delta X]^2\} \qquad 3.10$$

Since an impervious boundary exists at L = 1 and M = 1 (Fig. 3.1) there is no hydraulic gradient; therefore

$$U(0,M,N+1) = U(2,M,N+1)$$

$$\qquad 3.11$$

$$U(L,0,N+1) = U(L,2,N+1)$$

At L = I and M = J a pervious boundary exists and as such no excess pore pressure is possible, therefore

$$U(I,M,N+1) = 0$$

$$U(L,J,N+1) = 0 \qquad 3.12$$

The initial average effective stress is calculated from K_o conditions. The initial excess pore pressure u_i due to the external load is calculated from Equation 2.7. The stress increment ratio is given by

$$\frac{(\text{initial average effective stress + initial excess pore pressure})}{(\text{initial average effective stress})}$$

The governing equation is converted into a set of simultaneous algebraic equations using finite differences. These equations can be solved by Gaussian elimination.

The same procedure can be adopted for the linear Equation 2.38 with appropriate initial conditions. Since both the equations are mathematically equal they should yield the same solution for the same problem. Unfortunately, the non-linear Equation 2.31 could be solved only for a low range of stress increment ratio (constant ratio up to 5) whereas the linear Equation 2.38 could be solved for any range of stress increment ratio. Where comparison of stress increment ratios is possible agreement is found.

Experience indicates that Equation 2.31 is not stable for the numerical procedure used herein. This is also true for the general governing Equation 2.30. In order to extend the range of solution, the linear equivalent must be solved.

CHAPTER IV

RESULTS AND DISCUSSION

Equation 2.31 represents the general problem of two-dimensional pore pressure dissipation of a strip load of width 2B, of uniform load intensity q, (Fig. 2.1) resting on a half-space. The effective angle of internal friction (ϕ') is considered as 25° and the submerged weight of soil (γ') as 55 pcf. For analysis, the non-linear Equation 2.31 is transformed to the linear equation 2.38 by defining $w = \log_{10} \theta'/\theta'_f$. Equation 2.38 also yields the progress of consolidation in two-dimensions and it may be solved for a given set of boundary conditions.

Due to the variation of stress increment ratio (DR) within the defined clay region $0 \leq x \leq \beta B$ and $0 \leq z \leq H$, the excess pore pressure will vary spatially at time t = 0. The initial condition is given by (from Equation 2.32a)

$$w(x,z,0) = \log_{10}(\theta'_i(x,z)/\theta'_f(x,z)) \qquad 4.1$$

$$= \log_{10}(1/DR(x,z))$$

The boundary conditions for the clay layer (only half the section need be considered for the analysis because of

symmetry) at any time $t > 0$ are

$$\frac{\partial w}{\partial x} = 0 \quad \text{at} \quad x = 0$$

$$\frac{\partial w}{\partial z} = 0 \quad \text{at} \quad z = H \qquad 4.2$$

$$w = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad x = \beta B$$

For the given initial and boundary conditions (Equations 4.1 and 4.2 respectively) Equation 2.38 can be solved as an ordinary diffusion equation using numerical finite difference methods such as ADI. The average degree of consolidation $\overline{U}$, along the vertical section passing through the centreline is given by

$$\overline{U} = 1 - \frac{\int u_t \, dz}{\int u_i \, dz} \qquad 4.3$$

where u_t denotes excess pore pressure at time t

and u_i denotes initial excess pore pressure.

The average degree of consolidation, $\overline{U}$, is calculated at each time step by integration making use of Simpson's rule for the even numbers of ordinates in the range of integration.

The boundary specified by $x = \beta B$ and $0 \leq z \leq H$ is studied to determine its effect on the computed results. The results of the relationship between average degree of consolidation $\overline{U}$ and time factor shown in Fig. 4.1, Fig. 4.2 and Fig. 4.3 indicate that a

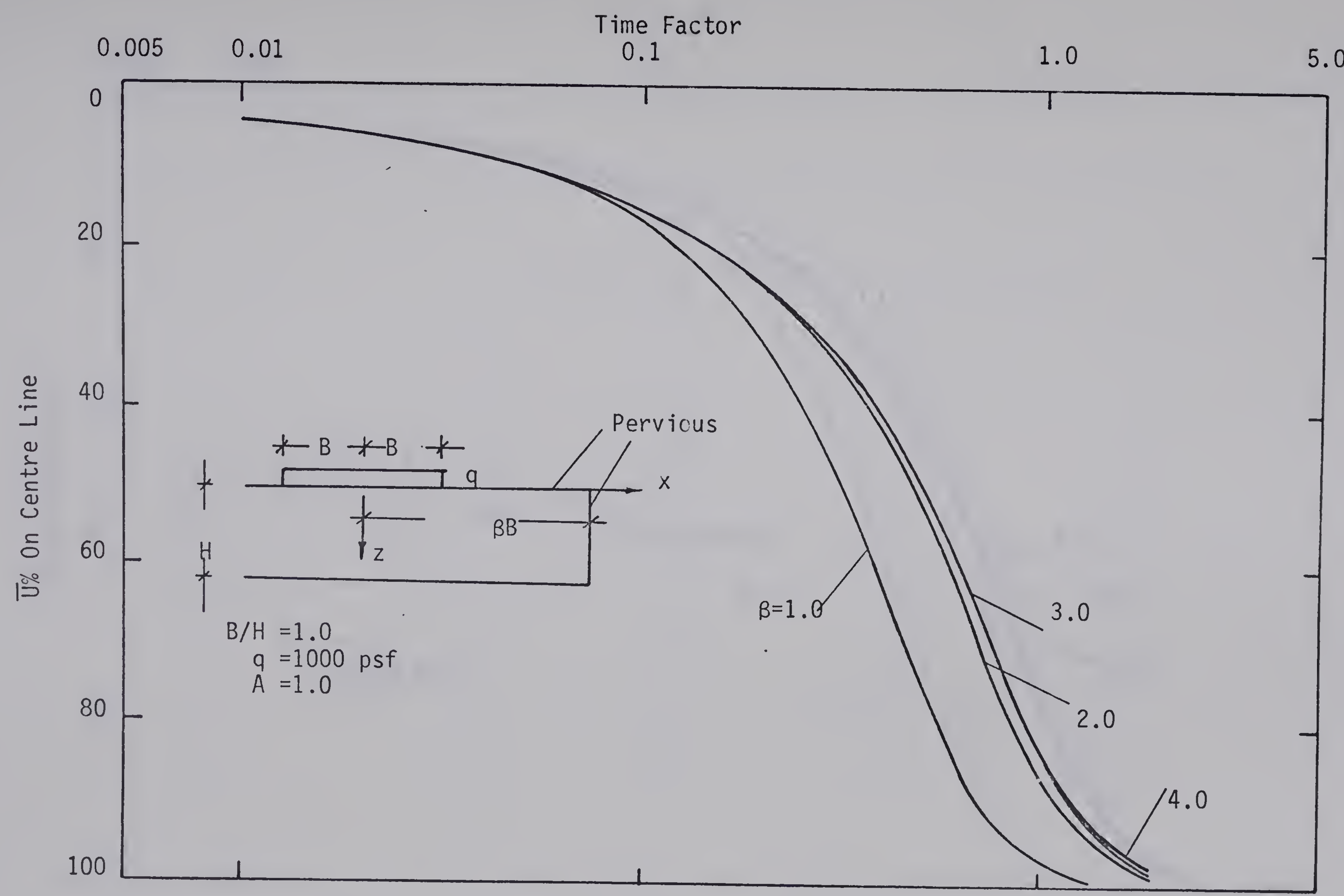


FIG. 4.1 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

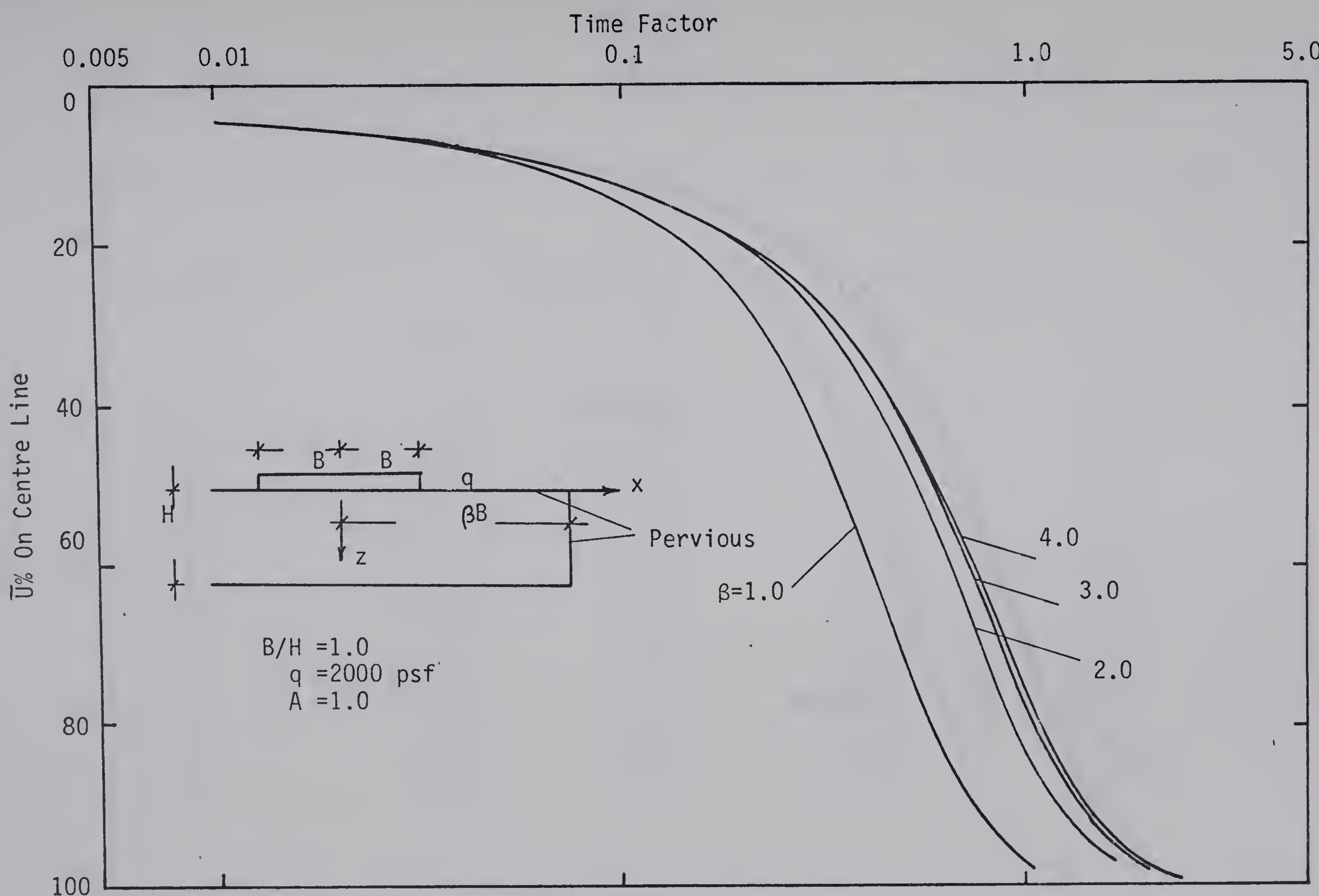


FIG. 4.2 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

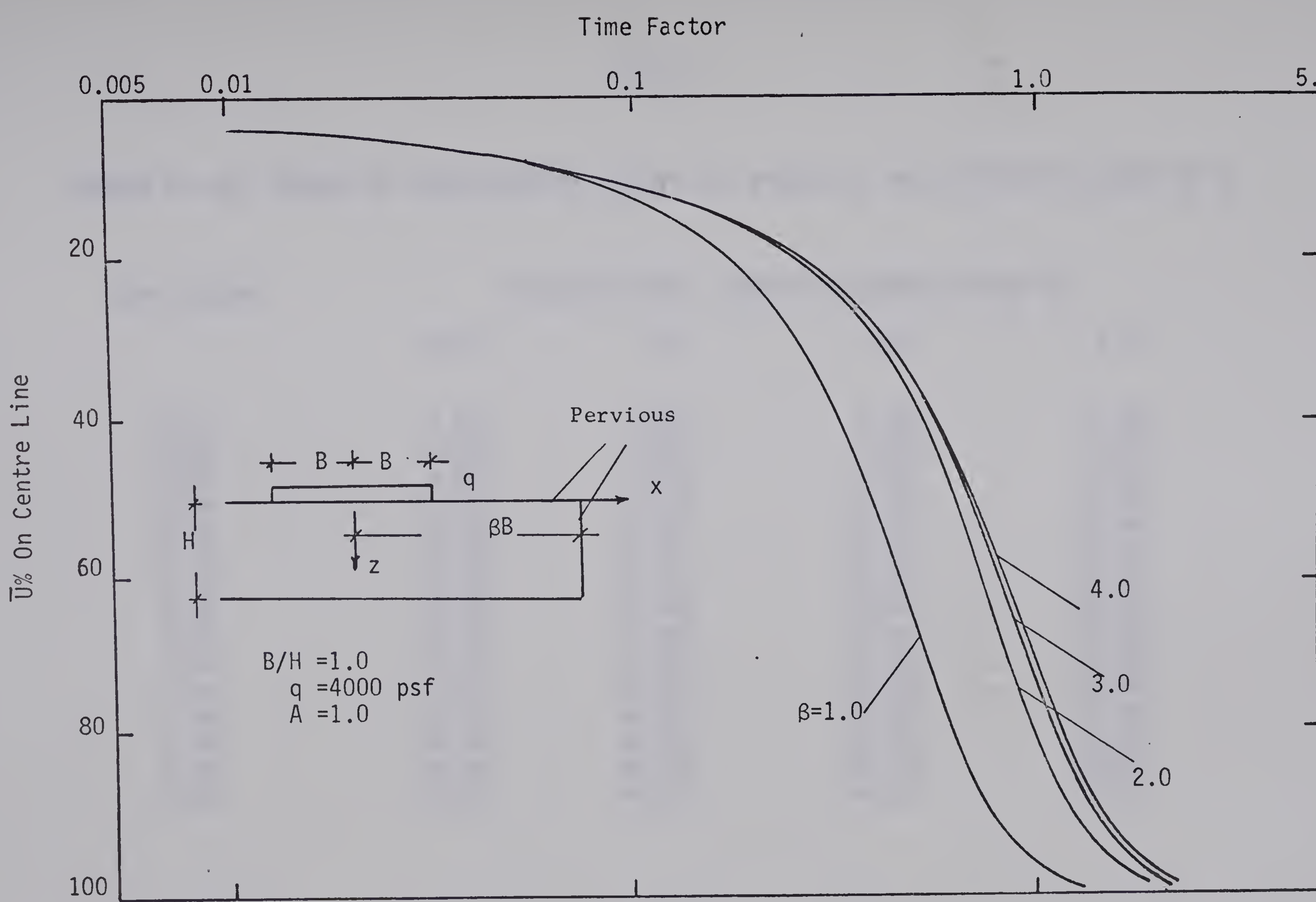


FIG. 4.3 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

TABLE 2

VALUES OF AVE. DEGREE OF CONSOLIDATION FOR A=1.0, q=1000 psf AND DIFFERENT VALUES OF β

TIME FACTOR	PERCENTAGE AVE. DEGREE OF CONSOLIDATION $\overline{U}$%			
	β=1.0	2.0	3.0	4.0
0.01	4.760	4.760	4.760	4.760
0.02	5.910	5.910	5.910	5.910
0.04	8.795	8.773	8.778	8.778
0.06	11.414	11.294	11.294	11.294
0.08	13.961	13.547	13.547	13.547
0.10	16.528	15.587	15.587	15.587
0.20	30.522	24.155	24.107	24.107
0.40	61.336	41.884	40.895	40.875
0.60	82.788	60.482	57.942	57.794
0.80	93.109	75.529	72.197	71.860
1.00	97.357	85.721	82.534	82.078
1.40	99.626	95.027	93.673	93.260
1.80	99.948	98.517	97.823	97.578
2.00	99.981	99.195	98.733	98.558
2.40	99.997	99.764	99.613	99.492

TABLE 3

VALUES OF AVE. DEGREE OF CONSOLIDATION FOR A=1.0, q=2000 psf AND DIFFERENT VALUES OF β

TIME FACTOR	PERCENTAGE AVE. DEGREE OF CONSOLIDATION $\bar{U}$%			
	β=1.0	2.0	3.0	4.0
0.01	4.395	4.377	4.370	4.367
0.02	5.319	5.276	5.262	5.254
0.04	7.723	7.571	7.532	7.512
0.06	9.923	9.566	9.498	9.464
0.08	12.066	11.331	11.233	11.186
0.10	14.238	12.913	12.786	12.727
0.20	26.512	19.491	19.172	19.062
0.40	56.828	34.383	32.470	32.231
0.60	80.193	52.800	48.243	47.639
0.80	91.967	69.591	63.584	62.479
1.00	96.903	81.820	76.061	74.657
1.40	99.561	94.210	90.884	89.695
1.80	99.939	98.266	96.801	96.139
2.00	99.977	99.059	98.130	97.672
2.40	99.997	99.724	99.368	99.166

TABLE 4

VALUES OF AVE. DEGREE OF CONSOLIDATION FOR A=1.0, q=4000 psf AND DIFFERENT VALUES OF β

TIME FACTOR	PERCENTAGE AVE. DEGREE OF CONSOLIDATION $\overline{U}$%			
	β=1.0	2.0	3.0	4.0
0.01	4.115	4.100	4.094	4.091
0.02	4.851	4.814	4.802	4.796
0.04	6.850	6.727	6.695	6.678
0.06	8.705	8.420	8.365	8.337
0.08	10.519	9.928	9.849	9.811
0.10	12.366	11.285	11.183	11.135
0.20	23.113	16.974	16.712	16.623
0.40	52.559	30.457	28.702	28.494
0.60	77.582	48.532	43.977	43.397
0.80	90.789	66.109	59.756	58.617
1.00	96.431	79.450	73.146	71.636
1.40	99.493	93.369	89.609	88.277
1.80	99.929	98.006	96.332	95.579
2.00	99.973	98.917	97.853	97.330
2.40	99.996	99.682	99.273	99.042

distance 4B(i.e. $\beta = 4$) is adequate in order to neglect the influence of the boundary. It is of interest to note that Krizek et al (1969) have drawn similar conclusions in their study. Computed results are given in Table 2, Table 3 and Table 4.

In their one-dimensional study, Davis and Raymond (1965) noted that the average degree of consolidation, $\overline{U}$, decreases with the increase of stress increment ratio for a given time factor, T_v. In the present study the shape of the curves in Fig. 4.4 indicate the same tendency. As the applied load increases, the stress increment ratio also increases with the result that the average degree of consolidation decreases with increasing load. Computed results in Table 5 show that the maximum variation in the average degree of consolidation for loads of 500 psf and 1000 psf is about 4.5% and the same variation is found for the loads 1000 - 2000 psf and 2000 - 4000 psf.

In this study it is assumed that Equation 2.7, as suggested by Henkel (1960), is a good approximation for representing the initial excess pore pressure u_i;

$$u_i = \theta + a\,\tau_{oct}$$

For an ideal saturated soil with an elastic skeleton, the initial excess pore pressure is given by Equation 2.6

$$u_i = \theta$$

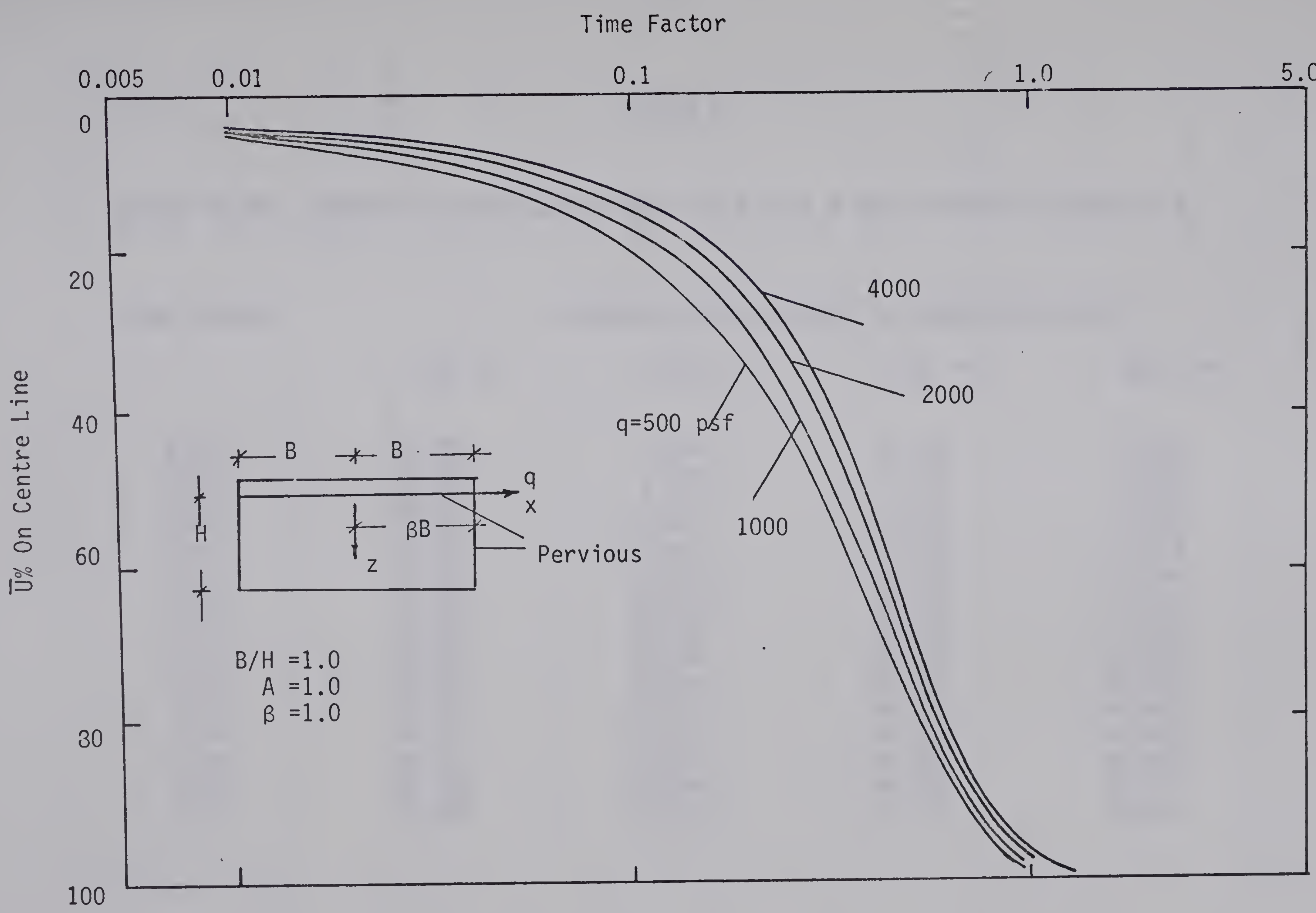


FIG 4.4 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

TABLE 5

VALUES OF AVE. DEGREE OF CONSOLIDATION FOR A=1.0, β=1.0 AND DIFFERENT VALUES OF q

TIME FACTOR	PERCENTAGE AVE. DEGREE OF CONSOLIDATION $\bar{U}$%			
	500 psf	1000 psf	2000 psf	4000 psf
0.01	5.219	4.760	4.395	4.115
0.02	6.627	5.910	5.319	4.851
0.04	10.060	8.795	7.723	6.850
0.06	13.170	11.414	9.923	8.705
0.08	16.194	13.961	12.066	10.519
0.10	19.226	16.528	14.238	12.366
0.20	35.073	30.522	26.512	23.113
0.40	65.929	61.336	56.828	52.559
0.60	85.288	82.788	80.193	77.582
0.80	94.184	93.109	91.967	90.789
1.00	97.781	97.357	96.903	96.431
1.10	98.637	98.375	98.094	97.800
1.20	99.164	99.003	98.830	98.649
1.30	99.488	99.389	99.283	99.172
1.40	99.687	99.626	99.580	99.540

To determine the effect on the relationship between average degree of consolidation $\overline{U}$ and time factor by using Equation 2.7 rather than Equation 2.6, calculations have been performed for the problem shown in Fig. 2.1.

It is found that the Henkel pore pressure parameter "a" has an insignificant effect on the rate of consolidation. This is clearly demonstrated by Fig. 4.5, Fig. 4.6 and Fig. 4.7. It appears reasonable to conclude that modifications to the general approach to take into account non-elastic effects on the initial pore pressures in terms of rate of consolidation are not worthwhile for practical purposes. The possible reason for this may be that the non-elastic effects constitute only about 25% of the elastic effects on the distribution of initial excess pore pressure. It has already been noted that the maximum variation in the average degree of consolidation for the range of loads employed is about 4.5%. The variation in average degree of consolidation is about 2% for an extreme range of "a" (Table 6).

Although the influence of non-elastic effects on the rate of consolidation in insignificant, however, the amount of settlement will be influenced.

It is known that the intensity of stress in the soil medium due to external load, decreases with increase of depth and with decrease of load. It controls the variation of initial excess pore pressure within the medium. Since the initial excess pore pressure at the mid-point of the bottom boundary is less, flow takes place

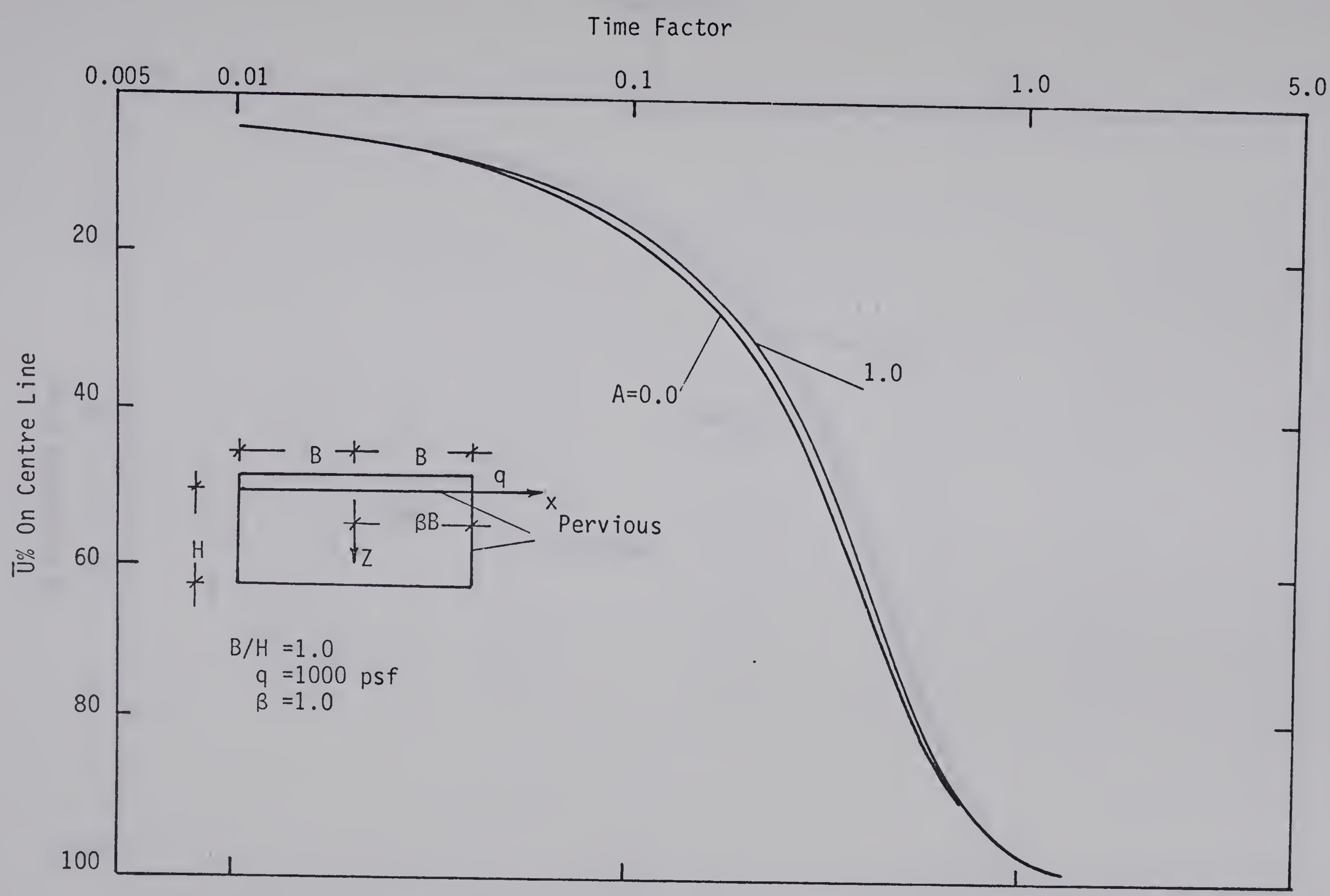


FIG. 4.5 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

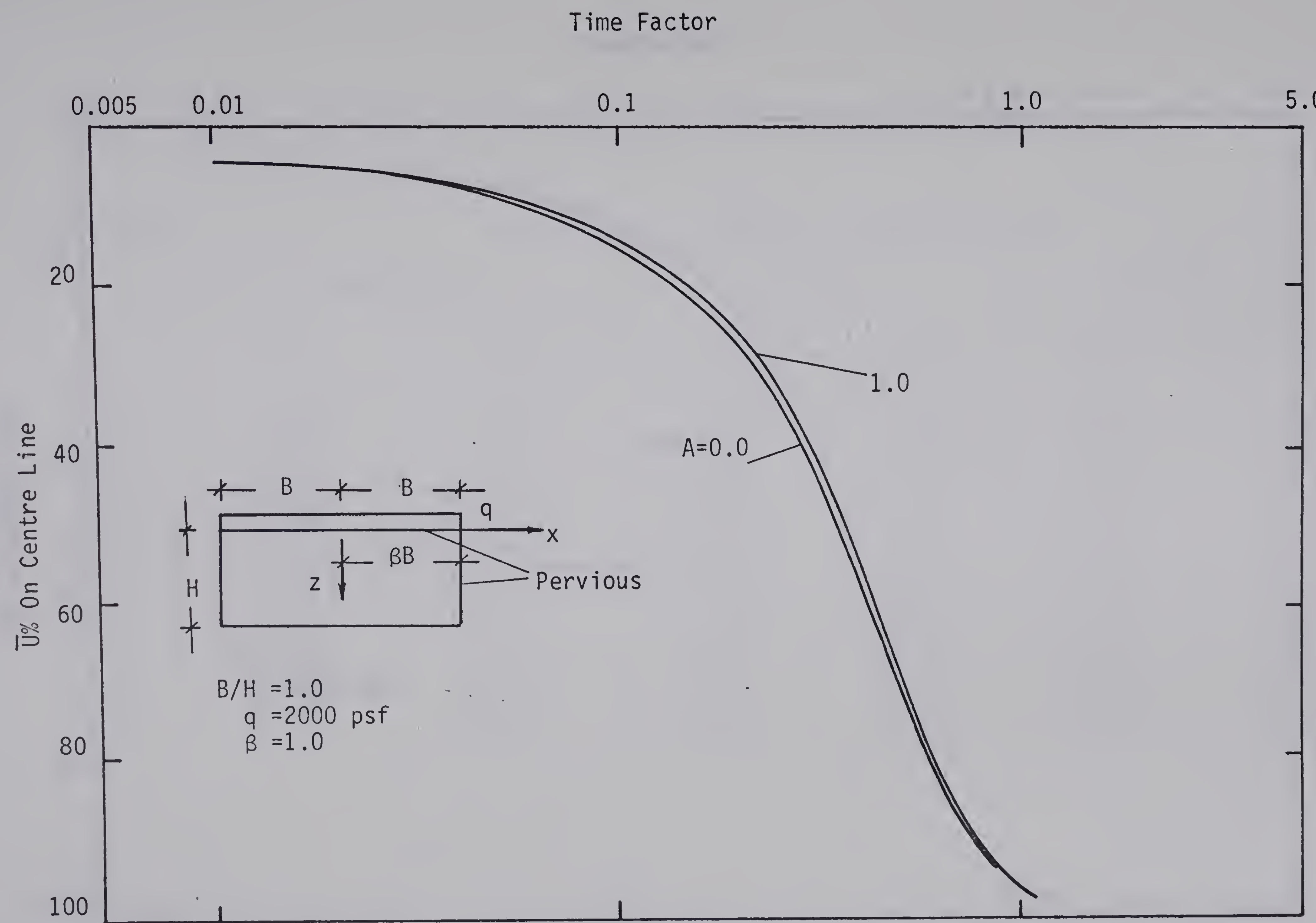


FIG. 4.6 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

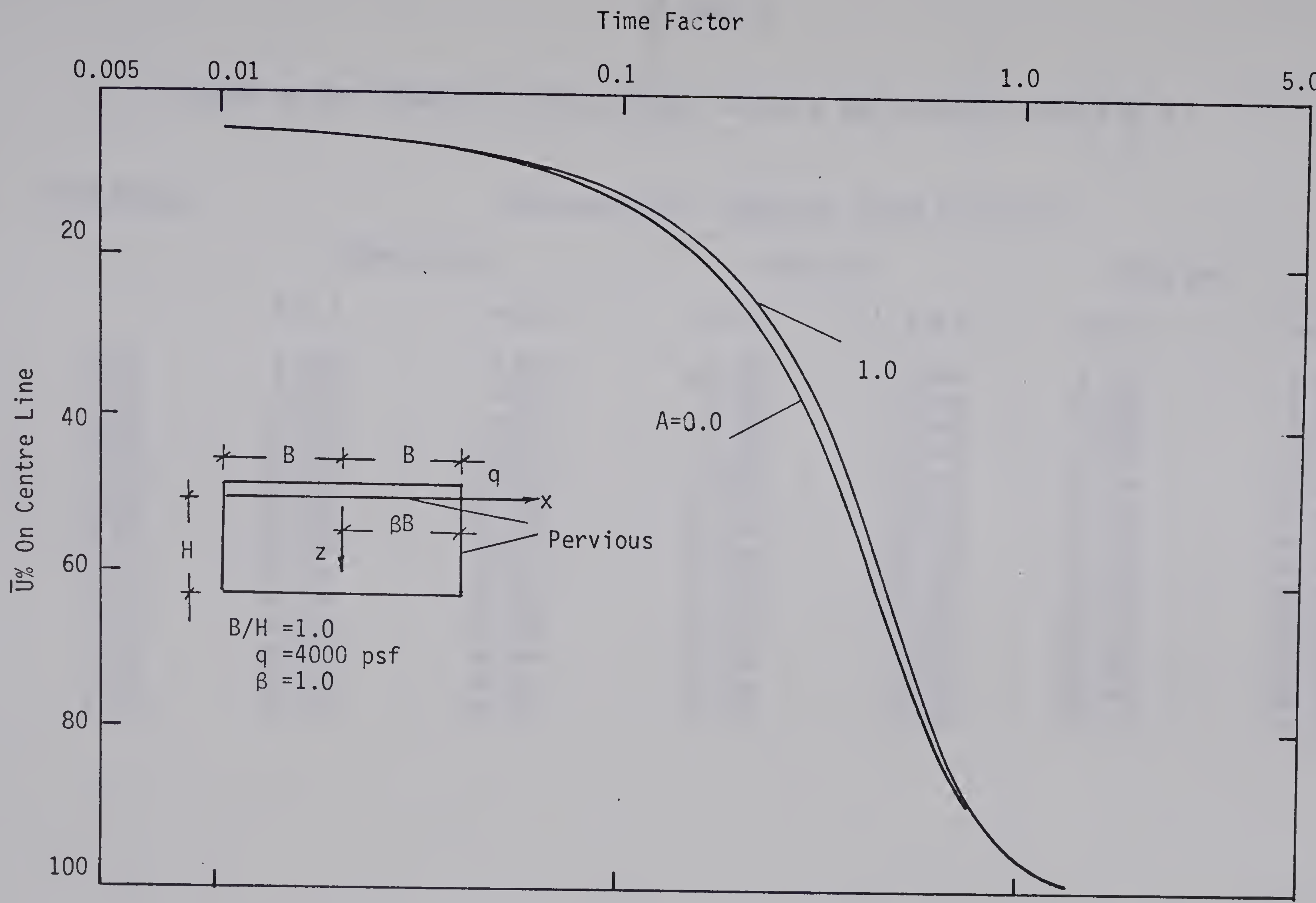


FIG. 4.7 AVERAGE DEGREE OF CONSOLIDATION *versus* LOG TIME FACTOR

TABLE 6

VALUES OF AVE. DEGREE OF CONSOLIDATION FOR β=1.0 AND DIFFERENT VALUES OF A

TIME FACTOR	PERCENTAGE AVE. DEGREE OF CONSOLIDATION $\bar{U}$%					
	LOAD=1000 psf		2000 psf		4000 psf	
	A=1.0	A=0.0	A=1.0	A=0.0	A=1.0	A=0.0
0.01	4.760	4.790	4.395	4.430	4.115	4.146
0.02	5.920	5.982	5.319	5.396	4.851	4.920
0.04	8.795	9.047	7.723	7.956	6.850	7.048
0.06	11.414	11.892	9.923	10.342	8.705	9.052
0.08	13.961	14.678	12.066	12.680	10.519	11.021
0.10	16.528	17.481	14.238	15.046	12.366	13.024
0.20	30.522	32.363	26.512	28.122	23.113	24.470
0.40	61.336	63.273	56.828	58.718	52.559	54.340
0.60	82.788	83.859	80.193	81.299	77.852	78.691
0.80	93.109	93.572	91.967	92.457	90.789	91.292
1.00	97.357	97.540	96.903	97.099	96.431	96.634
1.10	98.375	98.488	98.094	98.215	97.800	97.926
1.20	99.003	99.091	98.830	98.904	98.649	98.726
1.30	99.389	99.353	99.283	99.329	99.172	99.219

toward the mid-point due to the difference in hydraulic gradient. This process increases the pore pressure from the initial value and decays as time progresses. The effect is illustrated in Fig. 4.8. It may be noted that the increase of pore pressure is less as load increases. For a load of 500 psf the increased pore pressure is about 4% while for 4000 psf it is less than 1%.

Even though the increase of pore pressure is within a tolerable range from the point of view of safety, it is of interest to note that it is possible for the phenomenon to occur. Some results are synthesized in Table 7. Figure 4.9, Fig. 4.10, Fig. 4.11 and Fig. 4.12 display lines of equal pore pressure at different stages of consolidation.

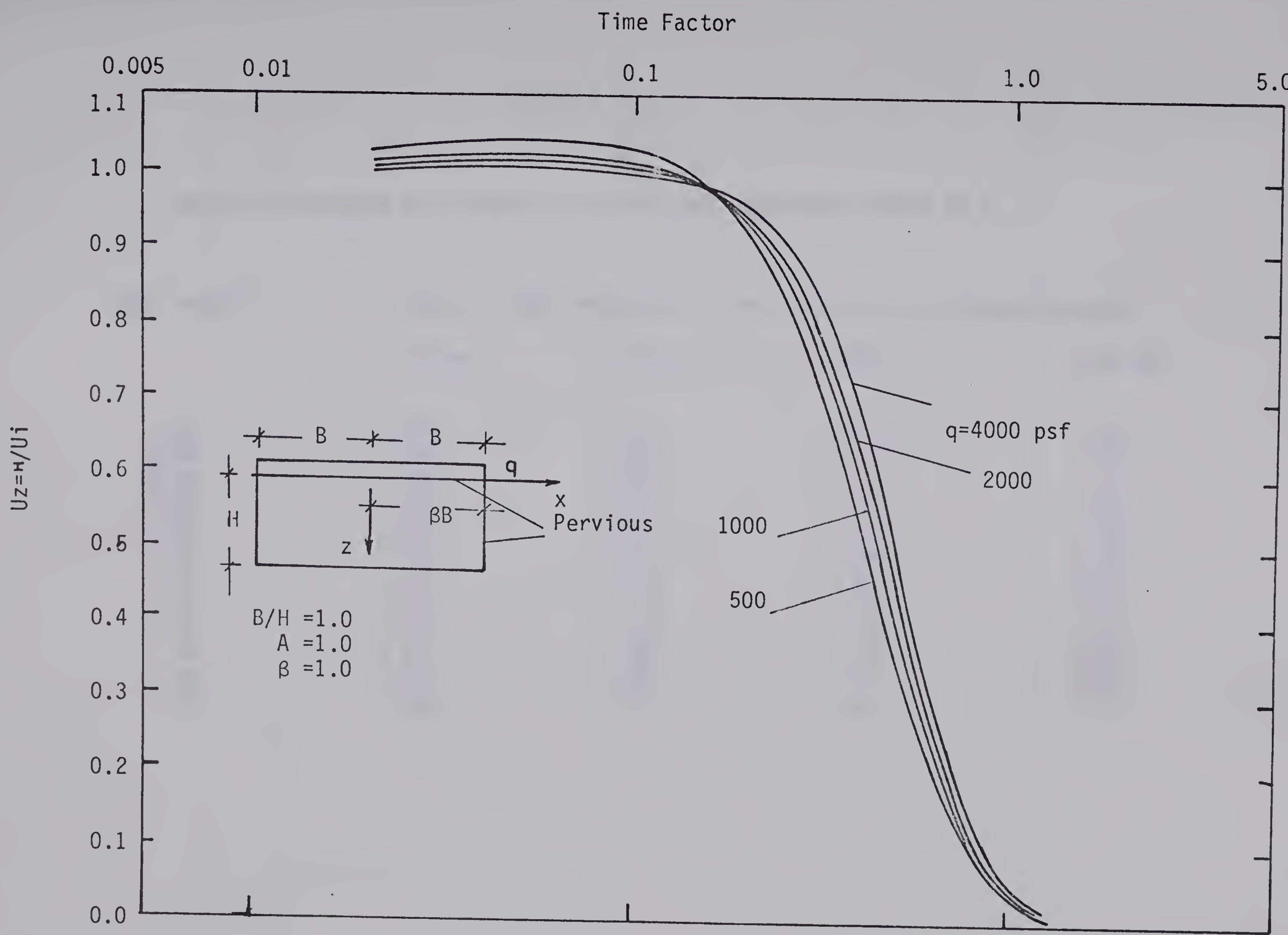


FIG. 4.8 MAXIMUM U/Ui *versus* LOG TIME FACTOR

TABLE 7

VALUES OF MAXIMUM U/Ui FOR A=1.0, β=1.0 AND DIFFERENT VALUES OF q

TIME FACTOR	RATIO OF PORE PRESSURE TO THE INITIAL PORE PRESSURE Uz=u/Ui			
	500 psf	1000 psf	2000 psf	4000 psf
0.02	1.027	1.014	1.007	1.004
0.04	1.038	1.019	1.010	1.005
0.06	1.043	1.022	1.011	1.005
0.08	1.041	1.020	1.009	1.004
0.10	1.032	1.014	1.006	1.002
0.20	0.917	0.930	0.945	0.959
0.40	0.530	0.578	0.627	0.675
0.60	0.238	0.270	0.304	0.340
0.80	0.095	0.110	0.126	0.143
1.00	0.037	0.043	0.049	0.056
1.10	0.023	0.026	0.030	0.035
1.30	0.008	0.010	0.011	0.013

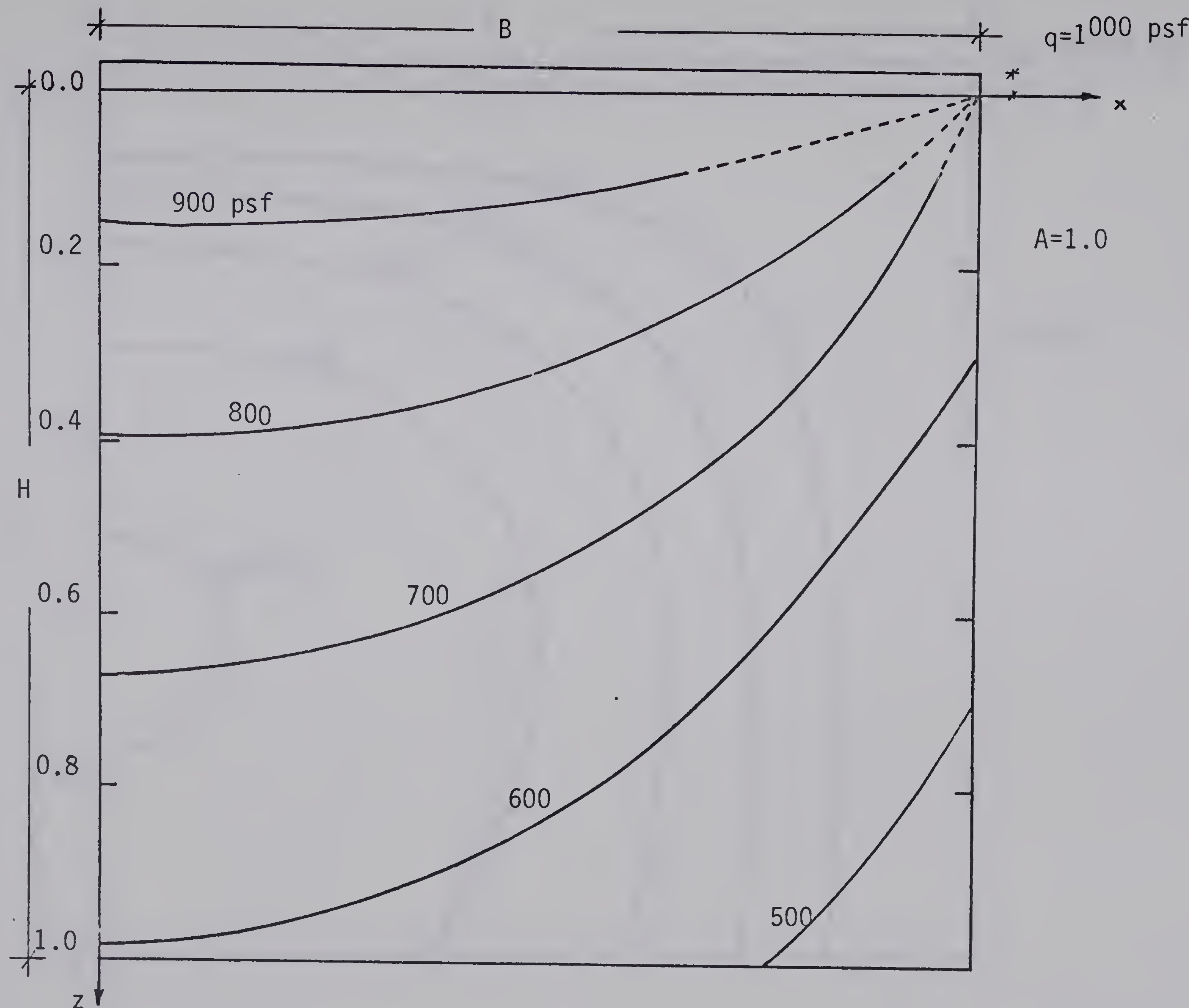


FIG. 4.9 LINES OF EQUAL PORE PRESSURE AT TIME FACTOR=0.0 ($\bar{U}$%=0.0)

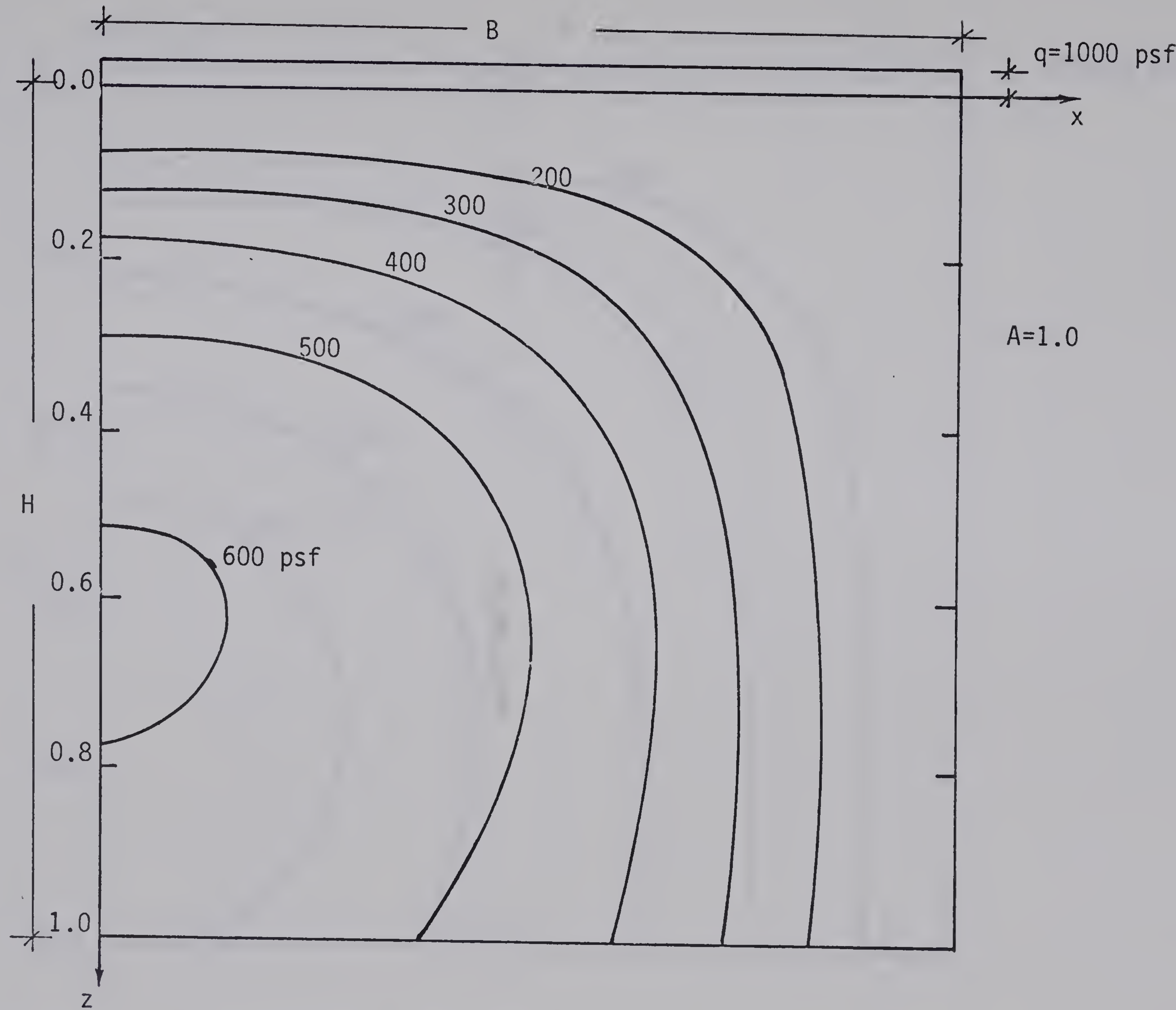


FIG. 4.10 LINES OF EQUAL PORE PRESSURE AT TIME FACTOR=0.2 ($\bar{U}$%=30.523)

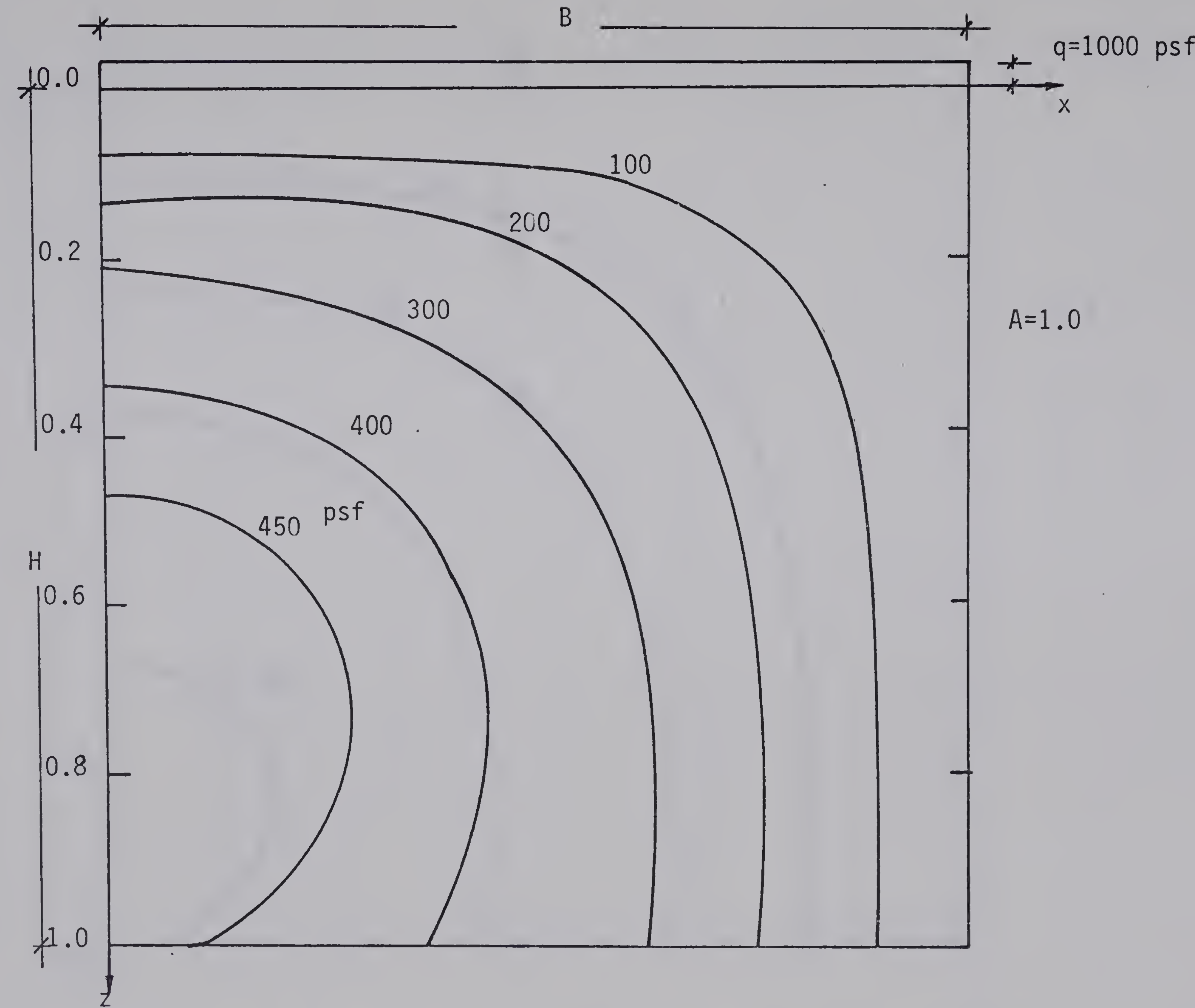


FIG. 4.11 LINES OF EQUAL PORE PRESSURE AT TIME FACTOR=0.4 ($\bar{U}$%=61.336)

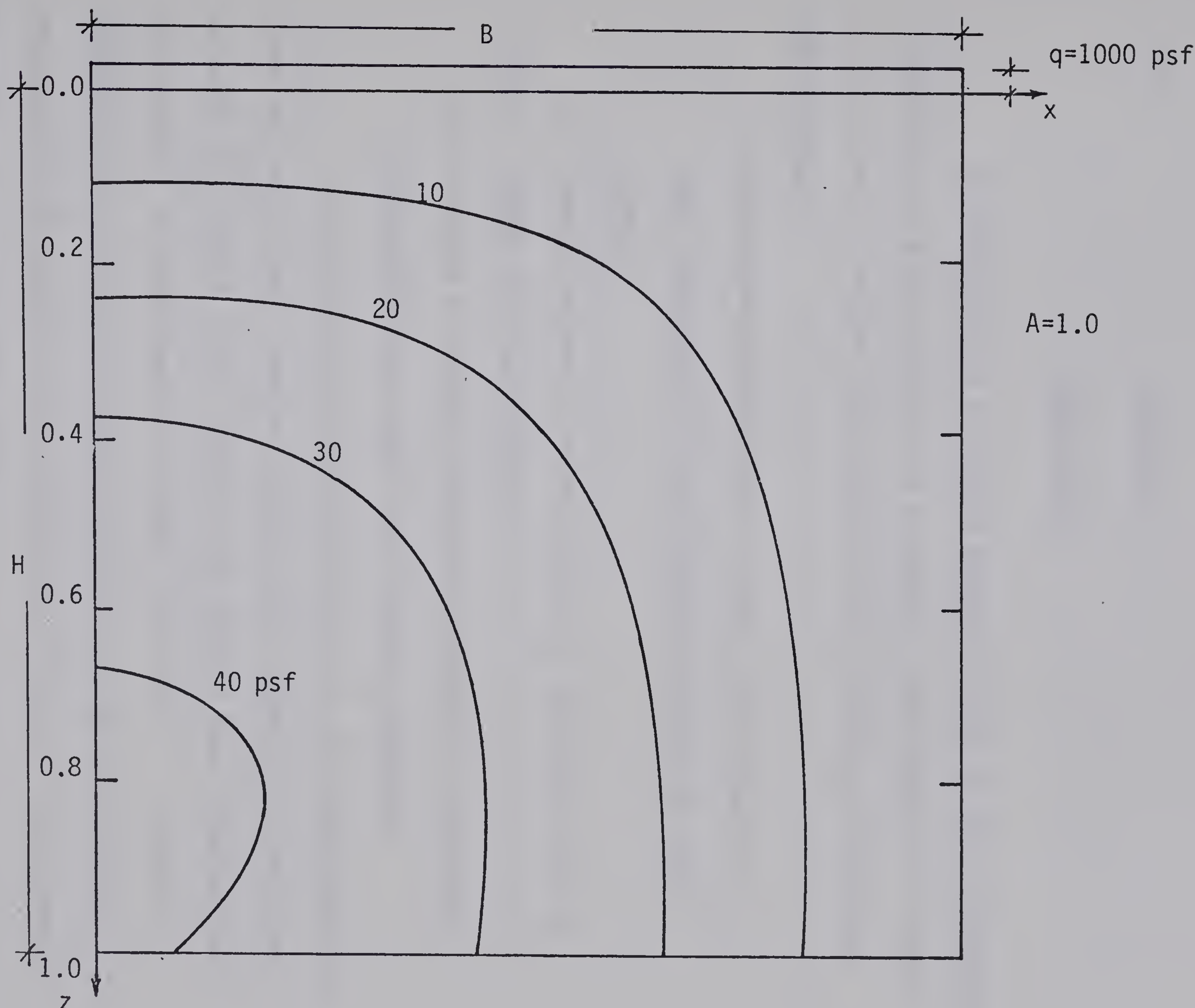


FIG. 4.12 LINES OF EQUAL PORE PRESSURE AT TIME FACTOR=0.9 ($\bar{U}$%=95.719)

CHAPTER V

CONCLUSIONS

On the basis of a limited computer study on the two-dimensional dissipation of pore pressures in a normally consolidated soil under a strip load of uniform load intensity, the following conclusions may be drawn:

1) The rate of dissipation of excess pore pressure is almost independent of pore pressures induced by shear stresses.
2) Average degree of consolidation, $\overline{U}$, decreases with increase of load intensity.
3) Increase of initial excess pore pressure is possible at the mid-plane after consolidation commences.

The conclusions presented herein, are, for the assumption that the total stress remains constant with respect to time as well as with space which is not true for the general case (constant only with time). As Schiffman et al (1969) point out ".....numerical techniques for solving non-linear problems are not well defined", regrettably the general case could not be solved with the numerical techniques adopted in this study.

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APPENDIX

COMPUTER PROGRAM AND USAGE

APPENDIX

The computer program presented herein describes the process of consolidation in two-dimensions (for thin layers) as given by the Davis and Raymond theory. The program calculates the ratio of pore pressure to the initial pore pressure at all nodal points and the average degree of consolidation at each time step. The program is in FORTRAN IV language and may be used directly on computers of Type 360/67.

The equation governing the dissipation of excess pore pressure in two-dimensions is given by Equation 2.38. The method of analysis by the ADI technique is described in Chapter III.

The program is applicable for a strip load of width 2B, of uniform load intensity q, imposed on the surface of a half-space. The program takes into consideration the stress increment ratio (DR) at all nodal points. The distribution of initial excess pore pressure depends on the stress increment ratio.

The entire program consists of a main and two subroutines. The main program reads the input data and sets up the initial and boundary conditions. The main program also regulates the channel to the subroutines by setting up appropriate conditional statements.

Subroutine JAM computes the stress components due to the external loading and calculates the stress increment ratio. Subroutine LEENA gives the ratio of pore pressure to initial pore pressure at all nodal points.

For the substitution employed (Equation 2.32a) the initial

condition at time zero is (i.e. N = 1)

$$w(L,M,1) = \log_{10} \frac{\theta'_i}{\theta'_f} = \log_{10} [1/DR(L,M)] \qquad A.1$$

For any time, say N (Fig. 3.1)

$$w(L,M,N) = \log_{10} \left\{\frac{\theta'_f - u(L,M,N)}{\theta'_f}\right\} \qquad A.2$$

Combining Equation A.2 and Equation 2.25, yields

$$\frac{u(L,M,N)}{u_i(L,M)} = DR(L,M) \{1 - e^{2.303\, w(L,M,N)}\}/[DR(L,M) - 1] \qquad A.3$$

Equation A.3 is the ratio of pore pressure to the initial excess pore pressure. Pore pressure at any node can be calculated by multiplying Equation A.3 with the corresponding initial excess pore pressure.

The average degree of consolidation on the basis of the centre-line integral method is calculated making use of Simpson's rule. The calculated values are printed out according to the out put formats.

Computer Program Usage

Input Data

To calculate the stress increment ratio and assign the boundary and initial conditions, the input data cards read the intensity of load, the soil parameters, space steps in both the di-

rections and the time factor steps.

The READ variables are:

GAMA, Q, DT, AB, I, J, K and DZ, DX.

The FORMAT is,

4F10.5, 3I10 and 2F10.5.

Output Information

The following information is computed and printed by the program at required time intervals:

1) the ratio of pore pressure to the initial excess pore pressure at all nodal points,
2) the time factor and the average degree of consolidation (one-dimensional integral) along the centre-line and
3) values of pore pressure at the required node.

```
C
C
C      **********************************************************************
C      DAVIS AND RAYMOND TWO-DIMENSIONAL ANALYSIS
C      **********************************************************************
C
C
C      THIS PROGRAM CALCULATES THE RATIO OF EXCESS PORE PRESSURE TO THE
C      INITIAL EXCESS PORE PRESSURE, PORE PRESSURE AND THE AVEREGE DEGREE
C      OF CONSOLIDATION BY ONE-DIMENSIONAL INTEGRAL METHOD
C
C
C      ******************************************************************
C
C      ANALYSIS FOR A STRIP LOAD OF WIDTH 2B OF UNIFORM INTENSITY,Q RESTING
C      ON THE HALF-PLANE
C
C      BA=HALF THE WIDTH OF LOADING
C      Q= FOOTING LOAD IN LBS./S.FT
C      SUBMERGED WEIGHT OF SOIL (GAMA) =55.0 PCF
C       PI=EFFECTIVE ANGLE OF INTERNAL FRICTION
C      H=DEPTH OF THE SECTION
C      AB=SKEMPTON PORE PR. PARAMETER----A---
C        ****************************************************************
C
C
C
C      TREATMENT IN A DIMENSIONLESS FORM
C
C      THIS IS DONE WITH RESPECT TO DEPTH,H SO THAT THE VARIATION
C      IN THE QUANTITY IN THE VERTICAL DIRECTION RANGES FROM 0.0 TO 1.0
C      AND IN THE HORIZONTAL DIRECTION RANGES FROM 0.0 TO BETA
C
C      TIME FACTOR IS DIMENSIONLESS AND IS EQUAL TO CV'*TIME/H**2
C
C
C
       COMMON U(11,41,21),DR(11,41),A(11),D(11),T(21),B(41),E(41),
      CGAMA,Q,BA,AB,DZ,DX,J,IA,JA,AZ(41),UZ
       COMMON/SQ/I,UJ(11,41)
C
C
 1000  READ(5,101,END=999) GAMA,Q,DT,AB,I,J,K
 101   FORMAT(4F10.5,3I10)
       READ(5,201) DZ,DX
 201   FORMAT(2F10.5)
       WRITE(6,541) GAMA,Q,DT,AB,I,J,K
 541      FORMAT(4X,'GAMA=',F10.5,2X,'Q=',F10.5,2X,'DT=',F10.5,2X,'AB=',
      CF10.5,2X,'I=',I10,2X,'J=',I10,2X,'K=',I10)
       WRITE(6,542) DZ,DX
 542      FORMAT(4X,'DZ=',F10.5,2X,'DX=',F10.5)
C
C      INITIAL AND BOUNDRY CONDITIONS
C
       BA=1.0
```

```
      BETA=(J-1)*DX/BA
      IA=I-1
      JA=J-1
      DO 10 N=1,K
      DO 10 L=1,I
      DO 10 M=1,J
      IF(N.EQ.1) GO TO 30
      IF(M.EQ.J.OR.L.EQ.I) U(L,M,N)=0.0
      GO TO 10
 30   IF(L.EQ.I) GO TO 31
      CALL JAM(L,M)
      U(L,M,N)=0.434*ALOG((1.)/DR(L,M))
      GO TO 10
 31   U(L,M,N)=0.0
 10   CONTINUE
C
C     PHYSICAL CONSTANTS
C
      RA=DT/(DZ)**2
      RB=DT/(DX)**2
      WRITE(6,110) RA,RB,BETA
 110  FORMAT(///4X,'RA=',F10.4,2X,'RB=',F10.4,2X,'BETA=',F10.4)
C
C     TWO DIMENSIONAL DISSIPATION BY  A  D  I  METHOD
C
      KK=K-2
      DO 121 MM=1,15
      DO 11 N=1,KK,2
C
C     DISSIPATION IN THE VERTICAL DIRECTION
C
      A(1)=2.*RA/(1.+2.*RA)
      AA=(1.-2.*RB)*U(1,1,N)+2.*RB*U(1,2,N)
      BB=1.+2.*RA
      D(1)=AA/BB
      DO 12 L=2,IA
      M=1
      A(L)=RA/(1.+2.*RA-RA*A(L-1))
      AA=(1.-2.*RB)*U(L,M,N)+2.*RB*U(L,M+1,N)
      BB=1.+2.*RA-RA*A(L-1)
      D(L)=(RA*D(L-1)+AA)/BB
 12   CONTINUE
      U(IA,1,N+1)=D(IA)
      DO 13 L=2,IA
      IS=I-L
      M=1
      U(IS,M,N+1)=D(IS)+A(IS)*U(IS+1,M,N+1)
 13   CONTINUE
      DO 14 M=2,JA
      L=1
      A(1)=2.*RA/(1.+2.*RA)
      AA=RB*U(L,M-1,N)+(1.-2.*RB)*U(L,M,N)+RB*U(L,M+1,N)
      BB=1.+2.*RA
      D(1)=AA/BB
      DO 15 L=2,IA
```

```
      A(L)=RA/(1.+2.*RA-RA*A(L-1))
      AA=RB*U(L,M-1,N)+(1.-2.*RB)*U(L,M,N)+RB*U(L,M+1,N)
      BB=1.+2.*RA-RA*A(L-1)
      D(L)=(RA*D(L-1)+AA)/BB
   15 CONTINUE
      U(IA,M,N+1)=D(IA)
      DO 16 L=2,IA
      IS=I-L
      U(IS,M,N+1)=D(IS)+A(IS)*U(IS+1,M,N+1)
   16 CONTINUE
   14 CONTINUE
C
C     DISSIPATION IN THE HORIZONTAL DIRECTION
C
      B(1)=2.*RB/(1.+2.*RB)
      AA=(1.-2.*RA)*U(1,1,N+1)+2.*RA*U(2,1,N+1)
      BB=1.+2.*RB
      E(1)=AA/BB
      DO 17 M=2,JA
      L=1
      B(M)=RB/(1.+2.*RB-RB*B(M-1))
      AA=(1.-2.*RA)*U(L,M,N+1)+2.*RA*U(L+1,M,N+1)
      BB=1.+2.*RB-RB*B(M-1)
      E(M)=(RB*E(M-1)+AA)/BB
   17 CONTINUE
      U(1,JA,N+2)=E(JA)
      DO 18 M=2,JA
      L=1
      IP=J-M
      U(L,IP,N+2)=E(IP)+B(IP)*U(L,IP+1,N+2)
   18 CONTINUE
      DO 19 L=2,IA
      M=1
      B(1)=2.*RB/(1.+2.*RB)
      AA=RA*U(L-1,M,N+1)+(1.-2.*RA)*U(L,M,N+1)+RA*U(L+1,M,N+1)
      BB=1.+2.*RB
      E(1)=AA/BB
      DO 22 M=2,JA
      B(M)=RB/(1.+2.*RB-RB*B(M-1))
      AA=RA*U(L-1,M,N+1)+(1.-2.*RA)*U(L,M,N+1)+RA*U(L+1,M,N+1)
      BB=1.+2.*RB-RB*B(M-1)
      E(M)=(RB*E(M-1)+AA)/BB
   22 CONTINUE
      U(L,JA,N+2)=E(JA)
      DO 23 M=2,JA
      IP=J-M
      U(L,IP,N+2)=E(IP)+B(IP)*U(L,IP+1,N+2)
   23 CONTINUE
   19 CONTINUE
   11 CONTINUE
C
C     CALCULATION OF THE RATIO OF PORE PRESSURE TO THE INITIAL PORE PRESSURE
C     AT ALL NODAL POINTS
C
      DO 24 N=1,K
```

```
      DO 24 L=1,IA
      DO 24 M=1,J
      EE=U(L,M,N)/(0.434)
      F=EXP(EE)
      U(L,M,N)=(DR(L,M)*(1.-F))/(DR(L,M)-1.0)
 24   CONTINUE
      DO 26 M=1,J
      IF(MM.GT.1) GO TO 27
      U(I,M,1)=1.0
      GO TO 26
 27   U(I,M,1)=0.0
 26   CONTINUE
C
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     OUT PUT PRINT
C     * * * * * * * * * ** * * * * * * * * * * *  * * * * * * * * ** ** * * *
C
C
      WRITE(6,541) GAMA,Q,DT,AB,I,J,K
      DO 85 N=1,K,20
      T(N)=DT*(N-1)
      WRITE(6,102) T(N)
 102  FORMAT(///50X,'TIME FACTOR=',F9.4)
      WRITE(6,103)
 103  FORMAT(//5X,'DISTANCE',40X,'RATIO OF THE PORE PRESSURE TO THE
     CINITIAL PORE PRESSURE')
      WRITE(6,105)
 105  FORMAT(/7X,'X/Y',8X,'0.0',7X,'0.1',7X,'0.2',7X,'0.3',7X,'0.4',7X,
     C'0.5',7X,'0.6',7X,'0.7',7X,'0.8',7X,'0.9',7X,'1.0'/)
      CALL LEENA(1,11,I,N)
      IF(J.LT.12) GO TO 85
      WRITE(6,306)
 306  FORMAT(/28X,'1.1',7X,'1.2',7X,'1.3',7X,'1.4',7X,'1.5',
     C7X,'1.6',7X,'1.7',7X,'1.8',7X,'1.9',7X,'2.0'/)
      CALL LEENA(12,21,I,N)
      IF(J.LT.22) GO TO 85
      WRITE(6,307)
 307    FORMAT(/28X,'2.1',7X,'2.2',7X,'2.3',7X,'2.4',7X,'2.5',
     C7X,'2.6',7X,'2.7',7X,'2.8',7X,'2.9',7X,'3.0'/)
      CALL LEENA(22,31,I,N)
      IF(J.LT.32) GO TO 85
      WRITE(6,308)
 308  FORMAT(/28X,'3.1',7X,'3.2',7X,'3.3',7X,'3.4',7X,'3.5',
     C7X,'3.6',7X,'3.7',7X,'3.8',7X,'3.9',7X,'4.0'/)
      CALL LEENA(32,41,I,N)
  85  CONTINUE
      WRITE(6,92)
 92   FORMAT('1')
C
C
C     AVE. DEGREE OF CONS. FROM CENTRELINE INTEGRAL METHOD
C     USING SIMPSON'S RULE
C     NO MODIFICATION IS REQUIRED SINCE THE NUMBER OF ORDINATES
C     IN CONSIDERATION IS EVEN
C
```

```
      WRITE(6,206)
  206 FORMAT(///50X,'TIME FACTOR',10X,'AVE.DEGREE OF CONSOLIDATION')
      DO 98 N=1,K,2
      IF(MM.GT.1) GO TO 90
      T(N)=DT*(N-1)
      M=1
      AZ(N)=U(1,M,N)-U(I,M,N)
      DO 106 L=2,IA,2
      AZ(N)=AZ(N)+4.*U(L,M,N)+2.*U(L+1,M,N)
  106 CONTINUE
      AZ(N)=AZ(N)*DZ/3.
      AZZ=AZ(1)
      GO TO 107
  90  T(N)=DT*((N-1)+(MM-1)*(K-1))
      M=1
      AZ(N)=U(1,M,N)-U(I,M,N)
      DO 96 L=2,IA,2
      AZ(N)=AZ(N)+4.*U(L,M,N)+2.*U(L+1,M,N)
  96  CONTINUE
      AZ(N)=AZ(N)*DZ/3.
  107 UZ=1.-AZ(N)/AZZ
      UZ=UZ*100.0
      WRITE(6,93) T(N),UZ
  93  FORMAT(//50X,F10.4,20X,F10.3)
  98  CONTINUE
C
C     CALCULATION OF EXCESS PORE PRESSURE AT THE MID-PLANE OF
C     BOTTOM SECTION
C
      WRITE(6,799)
  799 FORMAT(//30X,'TIME FACTOR',10X,'RATIO  U/U(INITIAL) ',10X,'PORE
     CPRESSURE(PSF)')
      DO 800 N=1,K,20
      IF(MM.GT.1) GO TO 900
      ZT=DT*(N-1)
      GO TO 901
  900 ZT=DT*((N-1)+(MM-1)*(K-1))
  901 UP=U(1,1,N)*UJ(1,1)
      WRITE(6,801) ZT,U(1,1,N),UP
  801 FORMAT(//30X,F7.3,17X,F9.3,20X,F10.3)
  800 CONTINUE
C
C
C
      DO 60 L=1,IA
      DO 60 M=1,J
      AA=((DR(L,M)-1.0)*U(L,M,K))/DR(L,M)
      U(L,M,1)=0.434*ALOG(1.-AA)
  60  CONTINUE
  121 CONTINUE
      WRITE(6,805)
  805 FORMAT('1')
      GO TO  1000
  999 STOP
      END
```

```
      SUBROUTINE JAM(L,M)
      COMMON U(11,41,21),DR(11,41),A(11),D(11),T(21),B(41),E(41),
     CGAMA,Q,BA,AB,DZ,DX,J,IA,JA,AZ(41),UZ
      COMMON/SQ/I,UJ(11,41)
C
C     CONSTANTS  -- ANGLE (PI')=25 DEG  --
C
      PI=3.1416*25./180.
      CO=1.-SIN(PI)
      ALPA=1.+0.5*(COS(PI)**2)
      ZETA=0.5*COS(PI)**2
C
C     STRIP LOADING   ----ELASTICITY APPROACH --
C
      UI=((1.+2.*CO)*GAMA*(I-L)*DZ)/(3.0)
      AA=ATAN(((M-1)*DX+BA)/((I-L)*DZ))
      BB=ATAN(((M-1)*DX-BA)/((I-L)*DZ))
      CC=2.*BA*(I-L)*DZ*(((M-1)*DX)**2-((I-L)*DZ)**2-BA*BA)
      DD=(((M-1)*DX)**2+((I-L)*DZ)**2-BA*BA)**2+4.*BA*BA*(((I-L)*DZ)**2)
      EE=CC/DD
      SZ=Q*(AA-BB-EE)/3.1416
      SX=Q*(AA-BB+EE)/3.1416
      TXZ=(4.*Q*BA*(M-1)*DX*(((I-L)*DZ)**2))/(3.1416*DD)
      FF=SQRT((SX-ZETA*(SZ+SX))**2+(ZETA*(SX+SZ)-SZ)**2+(SZ-SX)**2+
     C6.*TXZ*TXZ)
      UJ(L,M)=(ALPA*(SZ+SX)/3.)+((AB-(1./3.))*FF/(SQRT(2.)*3.))
      DR(L,M)=(UI+UJ(L,M))/UI
      RETURN
      END


      SUBROUTINE LEENA(J1,J2,I,N)
      COMMON U(11,41,21),DR(11,41),A(11),D(11),T(21),B(41),E(41),
     CGAMA,Q,BA,AB,DZ,DX,J,IA,JA,AZ(41),UZ
      DO 80 L=1,I
      IL=I-L
      AL=DZ*IL
      IL=IL+1
      IF(J1.GT.10) GO TO 272
      WRITE(6,166) AL,(U(IL,M,N),M=J1,J2)
  166 FORMAT(5X,F6.2,5X,11(F6.2,4X))
      GO TO 80
  272 WRITE(6,107) AL,(U(IL,M,N),M=J1,J2)
  107 FORMAT(5X,F6.2,15X,10(F6.2,4X))
   80 CONTINUE
      RETURN
      END
```